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ATOMS, SPECTRA, RADIATION

Simple Quantum Systems as a Source of Coherent Information

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Abstract—A set of very important simple quantum systems is analyzed from the standpoint of the amount of coherent information that is accessible when information channels corresponding to the systems are used. It is shown that for simple quantum models the coherent information can be calculated and used for estimating the potential possibilities of the corresponding quantum channel as a source of physical information in experiments associated with the effects of the coherence of quantum states. The following physical models are studied: a two-level atom in a laser radiation field, an aggregate of two two-level subsystems in a multilevel atom (hydrogen), a system of two two-level atoms in the process of joint quantum-deterministic evolution and under the action of transformations of quantum measurement and quantum duplication, as well as one and two two-level atoms in the process of emission. *© 2000 MAIK "Nauka/Interperiodica".*

1. INTRODUCTION

Finding a completely quantum analog of Shannon's quantitative measure of information [1] that would satisfy the corresponding quantum coding theorem, i.e., guarantee transmission along a quantum channel with a fixed information capacity irrespective of the physical nature of the channel, has for a long time remained a central unsolved problem of quantum information theory. The solution of this problem is given in [2, 3] using the concept of coherent information

$$I_c = S_{\text{out}} - S_{\text{e}}, \qquad (1)$$

where S_{out} describes the quantum entropy of the output variables of the channel and S_{e} is the exchange entropy, taken from a reservoir. If the measure I_c is positive, then expressed in qubits it gives the logarithm of the dimension of the Hilbert space, all states of which can be transmitted with probability $p = 1$ in the limit $N \longrightarrow \infty$ for long ergodic ensembles. In the opposite case, when the exchange entropy is greater than the output entropy and, correspondingly, the noise introduced by the channel completely nullifies the input information, we take $I_c = 0$.

There is every reason to expect that in application to physics coherent information will play a much larger role than Shannon's information. While in classical physics the information capacity of channels, arising in the process of a physical measurement, ordinarily can also be estimated without special calculations, at least in order of magnitude, this is far from being the case in the quantum situation. Analysis of the potentially accessible quantum information in the formulation of experiments in the newest directions of physics, associated with quantum computations, problems of quantum communication and quantum cryptography [4, 5], where the measure of coherent information of the physical channel used determines the potential information content of the data obtained, is especially important. However, in order to apply the concept of coherent information to physical systems the corresponding channel in the form of a superoperator transformation $\mathcal{C}$ must be specified for each system considered and the required quantum calculations, which, as a rule, are quite nontrivial, must be performed. It is shown in the present paper that this can be done, at least, for the most important simple quantum systems studied. The analysis is performed for systems of various physical nature, including channels with qualitatively different nature of the input and output of the type of atom in the electromagnetic field of the vacuum. The classification of the types of quantum channels considered, coupling two quantum systems, is given in Fig. 1.[1] The types of two-moment channels studied, where the information is transmitted from a state at an earlier moment $t = 0$ to a state at a later moment $t > 0$, must be supplemented by the corresponding single-moment analogs, in which information at the output concerning the state of the input at the same moment in time is considered. The first class is most closely associated with the problems of quantum communication and quantum measure-

[1] The specific limitations associated with the causality principle and due to the spatial localization of the systems 1 and 2 are important only for the channels $1 \longrightarrow 2$ and $1 \longrightarrow (1 + 2)$. The analysis performed below of a system of two atoms interacting via a radiation field requires that the relativistic retardation of the signal be taken into account in order to give a correct description of the dynamics at short times.

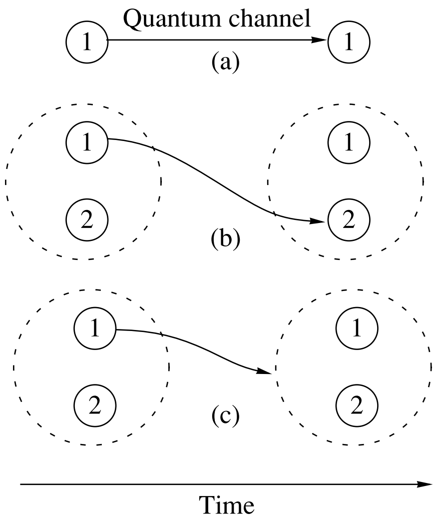

Fig. 1. Classification of possible quantum channels coupling two quantum systems: (a) 1 ⟶ 1—information is transferred from the initial state of a system to its final state; (b) 1 ⟶ 2—information is transferred from the subsystem 1 of the system (1 + 2) to the subsystem 2; (c) 1 ⟶ (1 + 2)—information is transferred from the subsystem 1 of the system (1 + 2) to the entire system (1 + 2).

ments, and the last class is associated with modern approaches to problems of quantum computations and quantum teleportation.

This paper is organized as follows. A description of the physical content of coherent information and the corresponding basic relations is given in Section 2. Section 3 is devoted to a description of the basic definitions and the technique of superoperator representations. The set of physical systems and the corresponding quantum channels is discussed in the next sections according to the classification presented in Fig. 1. The exchange of coherent information between the quantum states of a two-level atom (TLA) in a resonant laser field in two different moments in time (Fig. 1a) is discussed in Section 4. The same type of channel (1 ⟶ 1) is analyzed in Section 5 for a multilevel system, consisting of two systems of sublevels, for the example of the hydrogen atom. Section 6 examines the exchange of coherent information between two different quantum systems. It includes exchange between (1) two TLA coupled by a unitary transformation (Fig. 1b), (2) two TLA coupled via the procedure of quantum measurement (Fig. 1b), (3) an arbitrary system and its duplicated formed as a result of the quantum duplication procedure (Fig. 1c), (4) TLA and the field of the electromagnetic vacuum (Fig. 1b), and (5) two TLA coupled via a photon field of the vacuum (Fig. 1b). The basic results of this work are summarized in the conclusions.

2. QUALITATIVE MEANING OF COHERENT INFORMATION AND ITS RELATION WITH SHANNON'S INFORMATION

The classical measure of Shannon's information with error-free transmission of all possible values of a quantity x, which assumes M values, is given by $I = \log_2 M$, which for the given choice of the base of the logarithm it is conventionally assigned a "bit" as the unit of measurement. If the transmitted values x have different probabilities and are described by the probability distribution $P(x)$, then the definition presented is applicable not directly to x but to an ergodic sequence x_k ($k = 1, \ldots, n$) of statistically independent copies of x with the probability distribution $P(x_1) \cdot \ldots \cdot P(x_n)$. In this case, asymptotically for $n \longrightarrow \infty$, the set of sequences of nonzero probability consists of $M_n = 2^{nS(P)}$ approximately equally probable values, and one symbol corresponds to information $(\log_2 M_n)/n = S(p)$, where $S(P) = -\sum P(x)\log_2 P(x)$ is the Boltzmann entropy. This result, which, specifically, plays a fundamental role in statistical physics, gives the basis for assigning the value $I = S(P)$ to the information obtained with error-free transmission of all values of x with probability distribution $P(x)$. If errors are possible in transmission, then such a nontrivial information transmission channel is described by a conditional probability distribution $P(y|x)$ of the values of the output variable y for a fixed value of the input variable x. In this case, for long ergodic sequences the specific error-free transmitted information is described by the reciprocal Shannon information:

$$\begin{aligned} I &= S(P_x) + S(P_y) - S(P_{xy}) \\ &= S(P_y) - \sum_x S[P(y|x)]P(x). \end{aligned} \quad (2)$$

Here P_x, P_y, and P_{xy} are, respectively, the probability distributions for the input x, output y, and the pair (x, y). The first relation in Eq. (2) indicates the symmetric (reciprocal) character of Shannon's information with respect to input and output. The second relation gives the information as the difference of the entropy of the output variable y and the average value of the entropy introduced by the channel into the value of y for the transmission of a given a symbol x. The meaning of the latter relation is most transparent for a channel in which the transmitted values x are represented in transmission by nonoverlapping subsets M_x of the values of the quantity $y \in \cup M_x$, i.e., the distortions reduce to scatter of the output variable y in the regions M_x. The transmitted information is described, in this case, as the difference of the total entropy of y and the average entropy of the subsets M_x.

The initial definition of the coherent information is the relation $I_c = \log_2 \dim H$, where H is the Hilbert space of the states of the input quantum system, all states of which are transmitted without distortions. The natural term for the unit of quantum information is the term “qubit,” corresponding to a two-level quantum system with dimension $\dim H = 2$, that is used in the theory of quantum computations. The fundamentally new element of the theory is the quantum character of the transmitted information, which is described by an arbitrary coherent superposition of the basic elements. If the statistical distribution of the input states is described by the density matrix $\hat{\rho}_{\text{in}}$, then on the basis of considerations similar to those described above, with error-free transmission of quantum states $\psi \in H$ the measure of quantum information is the von Neuman entropy $I_c = S(\hat{\rho}_{\text{in}})$ where

$$S(\hat{\rho}) = -\mathrm{Tr}\hat{\rho}\log_2\hat{\rho}$$

is the direct operator generalization of the expression for Boltzmann’s classical entropy. The simplest channel implementing error-free transmission of information is, for example, the dynamical quantum evolution of a closed system considered at two moments in time, $t = 0$ and $t \geq 0$.

For a quantum channel with distortions the input state is represented as a linear transformation of the input state $\hat{\rho}_{\text{out}} = \mathscr{C}\hat{\rho}_{\text{in}}$. The superoperator $\mathscr{C}$ of the channel is analogous to the conditional probability distribution $P(y|x)$, considered above, of a classical channel. The quantum generalization of the Shannon definition (2) is constructed on the basis of the second relation, in which the first term—the quantum entropy of the output—has a unique quantum generalization in the form of the corresponding von Neuman entropy. The second term, describing the entropy introduced by the channel—the so-called exchange entropy S_e—should give in the quantum case with error-free transmission, i.e., for the identity superoperator $\mathscr{C} = \mathscr{I}$, a zero quantity, and for a pure state at the input (analog of the classical deterministic state) it should be identical to the entropy at the output, which in this case is determined only by the entropy introduced by the channel. These requirements can be satisfied by considering instead of the input quantum system its expansion $H \otimes H'$, where the variables H' do not interact with the channel variables, but rather the state $\hat{\rho}_P$ in the aggregate system is pure and such that after averaging it gives the initial state $\hat{\rho}_{\text{in}}$ [2]. This procedure of replacing the initial quantum system is called “purification” of the mixed quantum state. The corresponding transformation, performed by the channel on the composite quantum system, has the form $\mathscr{C} \otimes \mathscr{I}$, where $\mathscr{I}$ corresponds to constancy of the variables of the additional system, and the resulting exchange entropy is identified with the entropy of the transformed composite system. The specific form of the purified state in $H \otimes H$, i.e., with the choice $H' = H$, is explicitly contained in the formula, obtained in [3], whence follows

$$\hat{\rho}_P = \sum_{ij} \sqrt{p_i p_j} |i\rangle\langle j| \otimes |i^*\rangle\langle j^*|, \tag{3}$$

where p_i, $|i\rangle$, and $\langle j|$ are the eigenvalues and the right/left eigenvectors of the density matrix $\hat{\rho}_{\text{in}}$, and $|i^*\rangle$ and $\langle j^*|$ denote the complex-conjugate vectors. The purified state is combined, therefore, from the input system and its “mirror image.”[2] The corresponding exchange entropy has the form

$$S_e = S(\hat{\rho}_\alpha), \tag{4}$$

where

$$\hat{\rho}_\alpha = (\mathscr{C} \otimes \mathscr{I})\hat{\rho}_P. \tag{5}$$

The transformation $\mathscr{C}$ in the information channel, in general can describe the transfer of information to the output system with a different Hilbert space of states $H_{\text{out}} \neq H$.

For physical applications it is important to give an adequate physical interpretation of the density matrix (5) introduced in [3] and the density matrix, determined here, of the purified state (3), which initially appear from the above-described mathematical considerations. The expression (3) describes the combined state of the system consisting of the input and the mirror image, from which the quantum-mechanical state of the system input–output appears after transmission along the channel. In the classical theory the conditional probability $P(y|x)$ of the output with fixed input and, simultaneously, averaging with the distribution $P(x)$ over the states of the input corresponds to the state (5). The conditional distribution is represented by the superoperator $\mathscr{C}$, and averaging over the input is represented in the structure of the wave function

$$\Psi_P = \sum \sqrt{p_i} |i\rangle|i^*\rangle,$$

corresponding to the purified state (3). This two-particle state is entangled, i.e., it does not reduce to a statistical mixture of density matrices of the type $|\psi_i\rangle|\varphi_i\rangle\langle\varphi_i|\langle\psi_i|$, corresponding to pure states in the form of direct products $|\psi_i\rangle|\varphi_i\rangle$ of single-particle states. Its purely quantum fluctuations reproduce the fluctuations of a mixed nature, which are described by the density matrix $\hat{\rho}_{\text{in}}$, determined in the first space in the direct product $H \otimes H$. Therefore the density matrix (5) describes the state of the input–output system, where actually the input is replaced by the mirror-conjugate representation (see footnote 2). It determines the

[2] Compared with [3], here the complex conjugate, necessary for invariance of representation under study relative to rotations in subspaces corresponding to degenerate eigenvalues of the density matrix, is added. For real matrices $\hat{\rho}_{\text{in}}$ with a nondegenerate spectrum, this refinement is not essential.

exchange entropy in the channel and, on the basis of its physical meaning, is qualitatively different from the standard one-time density matrix, since the corresponding nonzero entropy appears only as a result of the transformation of the input state accompanying transmission along the channel. In the absence of distortions in the channel, in contrast to the standard two-particle density matrix, it always corresponds to a pure state and zero entropy.

3. BASIC DEFINITIONS AND THE SUPEROPERATOR REPRESENTATION TECHNIQUE

For purposes of the present paper, it is especially effective to use a combination of the technique of symbolic and matrix representation of superoperators [6]. The most general representation of a superoperator transformation is introduced by the symbolic expression

$$\mathcal{C} = \sum \hat{s}_{kl}\langle e_k| \odot |e_l\rangle, \tag{6}$$

where the substitution symbol $\odot$ must be replaced by an operator of the transformed physical quantity or the density matrix, while e_k describe an arbitrary vector basis in Hilbert space H where the transformed operator is defined. To describe physically realizable transformations of the density matrix $\hat{\rho}$, the operators $\hat{s}_{kl}$ must satisfy the positivity condition[3] of the block operator $\hat{S} = (\hat{s}_{kl})$ and the orthonormality condition

$$\mathrm{Tr}\hat{s}_{kl} = \delta_{kl}, \tag{7}$$

which ensures the required normalization for all normalized operators $\hat{\rho}$ with $\mathrm{Tr}\hat{\rho} = 1$.

Using the symbolic representation (6), it is possible to obtain the corresponding expression for the product of the superoperators $\mathcal{C}_1$ and $\mathcal{C}_2$, whence it is possible to give a symbolic representation of the superoperator algebra. For the case $\hat{s}_{kl} = |k\rangle\langle l|$ we obtain a representation of the identity superoperator $\mathcal{I}$, and for $\hat{s}_{kl} = |k\rangle\langle k|\delta_{kl}$ we obtain the representation of the quantum reduction superoperator

$$\mathcal{R} = \sum |k\rangle\langle k| \odot |k\rangle\langle k|.$$

The case $\hat{s}_{kl} = \delta_{kl}$ describes the superoperator of taking the trace $\mathrm{Tr}\odot$, which is a linear functional in the space of density matrices. The correspondence between the matrix form $S = (S_{mn})$ of the representation of the superoperator $\mathcal{C}$ in the orthonormal basis $\hat{e}_k$ and the representation (6) is given by the relation

$$\hat{s}_{kl} = \mathcal{C}(|k\rangle\langle l|) = \sum_{mn} S_{mn}\langle l|\hat{e}_n|k\rangle\hat{e}_m, \tag{8}$$

whose validity can be easily checked after substituting into the expression (6) and comparing with the standard definition of the matrix elements by means of the relation

$$\mathcal{C}\hat{e}_n = \sum_m S_{mn}\hat{e}_m.$$

The exchange entropy in the expression (1) for coherent information is determined by the relation (4), where the combined density matrix $\hat{\rho}_\alpha$ of the input–output variables is described in accordance with [3] and Eq. (5) by the relation

$$\hat{\rho}_\alpha = \sum_{ij} \mathcal{C}(|\rho_i\rangle\langle\rho_j|) \otimes |\rho_i^*\rangle\langle\rho_j^*|. \tag{9}$$

Here $|\rho_i\rangle = \hat{\rho}_{\mathrm{in}}^{1/4}|i\rangle$ are the transformed eigenvectors of the input density matrix

$$\hat{\rho}_{\mathrm{in}} = \sum p_i|i\rangle\langle i|,$$

$|\rho_i^*\rangle$ are the complex conjugates of $|\rho_i\rangle$, and $\mathcal{C}$ is the input–output transformation superoperator, so that $\hat{\rho}_{\mathrm{out}} = \mathcal{C}\hat{\rho}_{\mathrm{in}}$ describes the density matrix of the output variables. Using the superoperator representation in the form (6) and the above-defined eigenvectors $|i\rangle$, the density matrix (9) becomes

$$\hat{\rho}_\alpha = \sum_{ij} (p_i p_j)^{1/4}\hat{s}_{ij} \otimes |\rho_i^*\rangle\langle\rho_j^*|, \tag{10}$$

where the operators $\hat{s}_{ij}$ are the states of the output variables. Both the input and output partial density matrices can be represented as traces over the corresponding additional system: $\hat{\rho}_{\mathrm{out}} = \mathrm{Tr}_{\mathrm{in}}\hat{\rho}_\alpha$, $\hat{\rho}_{\mathrm{in}}^* = \mathrm{Tr}_{\mathrm{out}}\hat{\rho}_\alpha$.

To describe exchange of coherent information between two quantum systems via the quantum channels, shown in Figs. 1b, 1c ($1 \longrightarrow 2$ or $1 \longrightarrow (1 + 2)$) the initial combined density matrix must be given in the form of a direct product $\hat{\rho}_{1+2} = \hat{\rho}_{\mathrm{in}} \otimes \hat{\rho}_2$, where $\hat{\rho}_{in} = \hat{\rho}_1$ and $\hat{\rho}_2$ describes the initial partial density matrices, where the first one describes the input and the second describes the output channel. For a channel of the type $1 \longrightarrow 2$ the output are states of the second system, which contain information about the initial state of the first system, if a certain transformation over both systems is satisfied.

The temporal dynamics of the composite system (1 + 2) is described by the superoperator $\mathcal{C}_{1+2}$, and the corresponding superoperator transformation of the channel $\hat{\rho}_{\mathrm{out}} = \mathcal{C}\hat{\rho}_{\mathrm{in}}$ can be written as

$$\mathcal{C} = \mathrm{Tr}_1\mathcal{C}_{1+2}(\odot \otimes \hat{\rho}_2),$$

where the trace is calculated over the final states of the first system. In terms of the representation (6) for the

[3] The operators $\hat{s}_{kl} \otimes \hat{1}$ must be introduced in order to check positivity completely [7].

composite system this transformation can be described as

$$\mathscr{C} = \sum_{k\kappa l\lambda}\sum_{n} \langle n|\hat{s}_{k\kappa,\, l\lambda}|n\rangle\langle\kappa|\hat{\rho}_2|\lambda\rangle\langle k| \odot |l\rangle, \qquad (11)$$

where the multiplicative basis $|k\rangle|\kappa\rangle$ is used, and the indices k and κ correspond to the first and second quantum systems. The operator coefficients $\hat{s}_{kl}$ in Eq. (6) now assume the form

$$\hat{s}_{kl} = \sum_{\kappa\lambda}\sum_{n} \langle n|\hat{s}_{k\kappa,\, l\lambda}|n\rangle\langle\kappa|\hat{\rho}_2|\lambda\rangle. \qquad (12)$$

Here $\mathscr{C}$ depends on the form of the combined dynamical transformation $\mathscr{C}_{1+2}$ and on the initial state $\hat{\rho}_2$ of the second system, and it maps the initial states of the first system into the final states of the second system.

Ordinarily, it is much easier to calculate the one-time amount of information, since the input–output density matrix is simply a one-time density matrix of the corresponding variables, which is calculated directly from the dynamical equations. For one system, the corresponding channel is described by the single superoperator $\mathscr{I}$ and the corresponding calculations are trivial: for the combined input–output density matrix (9) we obtain the pure state

$$\hat{\rho}_\alpha = \sum_i |\rho_i\rangle|\rho_i^*\rangle \sum_j \langle\rho_i^*|\langle\rho_j|,$$

and the corresponding exchange entropy $S_e = 0$ and coherent information $I_c = S_{\text{out}} = S_{\text{in}}$. For two systems, where the input–output density matrix is a combined density matrix $\hat{\rho}_{1+2}$, the corresponding coherent information in the system 2 about the system 1 at the time t is

$$I_c(t) = S[\hat{\rho}_2(t)] - S[\hat{\rho}_{1+2}(t)].$$

When the dynamics is described by a unitary transformation and the initial state of the second system is pure, all eigenstates $|i\rangle$ of the first system transform into the corresponding set of orthogonal states $\Psi_i(t)$ of the composite system $(1 + 2)$, so that the combined entropy remains unchanged, and the coherent information becomes

$$I_c(t) = S[\hat{\rho}_2(t)] - S[\hat{\rho}_1(0)].$$

If the initial state of the first system is also pure, then we obtain simply $I_c(t) = S[\hat{\rho}_2(t)]$. For a TLA this gives $I_c = 1$ qubit, if the maximally entangled state is attained in a system of two qubits.

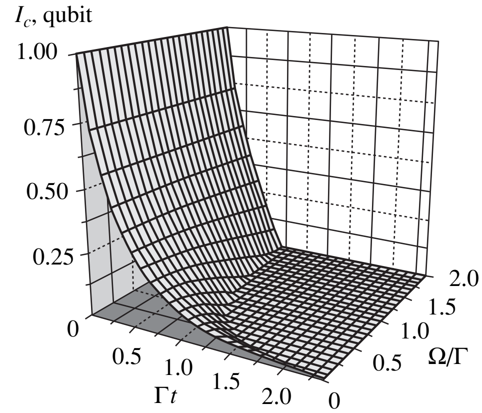

Fig. 2. Coherent information transferred from the initial state of a TLA at $t = 0$ state at the moment $t > 0$ as a function of the dimensionless time Γt and the Rabi frequency Ω/Γ.

4. TWO-LEVEL ATOM IN A RESONANT LASER FIELD

We shall consider the exchange of coherent information between the states of a TLA in a resonant laser field at two different times (Fig. 1a).

An example of a channel of this type was examined in [3], where only pure dephasing in the absence of an external field was studied. In the presence of a field and other relaxation mechanisms, the calculation of coherent information on the basis of the Markov approximation can be performed in the most general form by calculating the combined density matrix (9) using the technique of matrix representation of dynamical superoperators. One question of interest is the form of the dependence of the coherent information on the applied resonant field.

An external field changes the characteristic decay rates of the initial state of a TLA, which are described by the real parts of the eigenvalues λ_k of the dynamic Liouville operator $\mathscr{L} = \mathscr{L}_r + \mathscr{L}_E$, where the Liouville operators $\mathscr{L}_r$ and $\mathscr{L}_E$ describe relaxation and interaction with an external field. Here we confine our attention to relaxation represented only by pure dephasing in combination with the action of a laser field. The corresponding Liouville matrix in the operator basis $\hat{e}_k = \{\hat{I}, \hat{\sigma}_3, \hat{\sigma}_1, \hat{\sigma}_2\}$ has the form [8]

$$L = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & \Omega \\ 0 & 0 & -\Gamma & 0 \\ 0 & -\Omega & 0 & -\Gamma \end{pmatrix}, \qquad (13)$$

where Γ describes the rate of decay of the phase in the absence of the field, Ω is the Rabi frequency, and $\hat{\sigma}_1$, $\hat{\sigma}_2$, and $\hat{\sigma}_3$ are the Pauli matrices. The eigenvalues of the matrix (13) have the form

$$\lambda_k = \{0, -\Gamma, -(\Gamma + \sqrt{\Gamma^2 - 4\Omega^2})/2, -(\Gamma - \sqrt{\Gamma^2 - 4\Omega^2})/2\}.$$

The laser field changes these quantities compared with their unperturbed values 0, –Γ, –Γ, and 0.

It is of interest to determine whether or not such a change in the decay rates results in a decrease of the decay rate of the coherent information, though from intuitive considerations it can be inferred beforehand that an information gain is possible only in the case of another effect related with the laser field—decrease of the relaxation parameters of the relaxation superoperator $\mathcal{L}_r$ itself [8–11].

Calculating the matrix of the dynamical superoperator $\mathcal{C} = \exp(\mathcal{L}t)$ and using the corresponding representation (6), we obtain an analytical expression for the combined density matrix (9) and then [using Eqs. (4) and (1)] we calculate the coherent information retained in the TLA at the time t relative to its initial state. The latter is chosen in the form of the density matrix $\hat{\rho}_0 = \hat{I}/2$ with maximum entropy $S(\hat{\rho}_{in}) = 1$ qubit. The computational results are displayed in Fig. 2. They are described by a threshold-type time dependence, typical for coherent information limited by coherence loss processes. In addition, it is clearly seen that the coherent information does not increase, and it even decreases somewhat with increasing field intensity, as described by the corresponding Rabi frequency.

The results presented demonstrate also the singularity of the first time derivative of the coherent information at time $t = 0$, which is a characteristic feature of the initial stage of its decay. Indeed, initially the input–output density matrix (9) of a TLA has the form of a pure state $\hat{\rho}_\alpha = \Psi\Psi^+$ with the input–output wave function $\Psi = \sum \sqrt{p_i}\,|i\rangle|i\rangle$. Its eigenvalues λ_k and the probabilities of the corresponding eigenvalues are all zero, except the one corresponding to Ψ. As a result of the singularity of the entropy function $\sum(-\lambda_k)\log_2\lambda_k$ at $\lambda_k = 0$, the derivative of the corresponding exchange entropy also possesses a logarithmic singularity.

Another interesting feature of the coherent information is the form of its dependence on the initial (input) state $\hat{\rho}_{in}$. If it were possible, it would make sense to choose it in the form of the characteristic Liouville operator

$$\hat{\rho}_{in} = \sum_{l=1}^{4} |k_{min}\rangle_l \hat{e}_l,$$

where $|k_{min}\rangle$ is the eigenvector corresponding to the minimum eigenvalue $|\mathrm{Re}\lambda_k| > 0$ of the matrix L. However, the vector $|k_{min}\rangle$ equals $\{0, (\Gamma + \sqrt{\Gamma^2 - 4\Omega^2})/2\Omega, 0, 1\}$, i.e., it describes an element of the linear subspace of operators with zero trace, since the first component is zero. Therefore the decay of coherent information cannot be decreased by decreasing the rate of decay of the coherence of the atomic state in a laser field.

5. EXCHANGE OF COHERENT INFORMATION BETWEEN TWO OPEN SUBSYSTEMS OF A SINGLE SYSTEM

Let us consider the quantum channel of the type 1 ⟶ 1 (Fig. 1a) between two open subsystems A and B of a single closed system $\{A, B\}$ with the Hilbert space of states $H_A + H_B$, where H_A and H_B are the Hilbert subspaces of the systems A and B, respectively, and the "+" sign is used to denote a linear union.

In classical information theory this situation corresponds to transfer of only the part $A \subset X$ of the values of the input random variable $x \in X$. The realization where the detector does not obtain any message also carries information and means that x belongs to the complement of A, $x \in \bar{A}$. This situation can be described by the corresponding transformation of the choice $\mathcal{C} = P_A + P_0(1 - P_A)$, where P_A is the projection operator from X onto the subset A, $P_Ax = x$ for $x \in A$ and $P_Ax = \varnothing$ (empty set) for $x \in \bar{A}$, while P_0 is the projection operator from X onto an independent single-point set X_0 and $P_0x = X_0$. This transformation corresponds to the classical reduction channel, which results in information losses, only if $\bar{A}$ does not consist of only one point. If A is only point, then it is possible to obtain a potential limit of information equal to 1 bit, because $\bar{A}$ replaces the second state of the bit, so that actually no information is lost.

In quantum mechanics the corresponding reduction channel is described by an obvious generalization of the classical selection operator—the selection superoperator

$$\mathcal{C} = \hat{P}_A \odot \hat{P}_A + |0\rangle\langle 0|\mathrm{Tr}(1 - \hat{P}_A) \odot (1 - \hat{P}_A), \quad (14)$$

where the state $|0\rangle$ is the quantum analog of the classical one-point set, which does not depend on all the other states. This transformation is positive and preserves the normalization of the density matrix, describing adequately the exchange of coherent information between open subsystems of a single system. The last term in Eq. (14) expresses conservation of normalization, provided that all states outside the set of B states are included. In our case these states are all included in the form of the projector $|0\rangle\langle 0|$, which does not take into account their coherence. In contrast to the classical one-bit case, for a TLA they do not carry any coherent information because of the complete loss of coherence.

Considering the coherent information transmitted from one part A to the part B of a system, whose state depends on time, we are dealing with a superoperator of this channel of the form

$$\mathscr{C}_{AB} = \mathscr{C}_B \mathscr{C}_0(t) \mathscr{C}_A, \quad \mathscr{C}_0(t) = U(t) \odot U^{-1}(t) \quad (15)$$

with a unitary temporal resolution operator $U(t)$ and selection superoperators $\mathscr{C}_A$ and $\mathscr{C}_B$ of the subsystems A, B. Here the selection superoperator $\mathscr{C}_A$ is presented only for the possibility of determining the complete superoperator of the channel irrespective of the form of the input density matrix. However, if the input density matrix $\hat{\rho}_{\text{in}}$ is determined only in the corresponding subspace H_A of the complete space H, this superoperator can be dropped.

Let us assume that the dynamical evolution of the system is given by a set of eigenstates $|k\rangle$ and the corresponding Bohr frequencies ω_k. Then, representing the projectors in terms of the corresponding input $|\psi_l\rangle$ and output $|\varphi_m\rangle$ wave functions, we obtain from Eq. (15) the representation of the temporal evolution specified in the form

$$\mathscr{C}_{AB}(t) = \sum_{ll' \in A} \Bigg[\hat{s}_{ll'}(t) + |0\rangle\langle 0| \times \sum_{m \notin B} \langle \varphi_m | \psi_l(t) \rangle \langle \psi_{l'}(t) | \varphi_m \rangle \Bigg] \langle \psi_l | \odot | \psi_{l'} \rangle, \quad (16)$$

$$\hat{s}_{ll'}(t) = \sum_{mm' \in B} \langle \varphi_m | \psi_l(t) \rangle \langle \psi_{l'}(t) | \varphi_{m'} \rangle | \varphi_m \rangle \langle \varphi_{m'} |,$$

$$|\psi_l(t)\rangle = \sum_k \exp(-i\omega_k t) \langle k | \psi_l \rangle | k \rangle.$$

Let us consider the case of an orthogonal choice of subsets of input/output wave functions, which is of special interest. Then, if there is only one common state $|\phi\rangle$ in the sets $|\psi_l\rangle$, $|\varphi_m\rangle$ and $U(t_0) = 1$ for a time t_0, we obtain the expression

$$\mathscr{C}_{AB}(t_0) = |\phi\rangle\langle\phi| \odot |\phi\rangle\langle\phi| + |0\rangle\langle 0| \sum_{\varphi_m \neq \phi} \langle \varphi_m | \odot | \varphi_m \rangle,$$

which means that the input system reduces to a classical bit of information, associated with the states $|\phi\rangle$ and $|0\rangle$, and no coherent information is transmitted into the system B. Nonetheless, it appears in the process of temporal evolution, provided that the eigenstates $|k\rangle$ of the operator $U(t)$ are different from the input/output states $|\psi_l\rangle$, $|\varphi_m\rangle$. Therefore, the information capacity of the channel is due to the quantum entanglement of the input and output on account of the corresponding contribution to the Hamiltonian systems.

To illustrate the exchange of coherent information in the channel of the type described, we shall consider

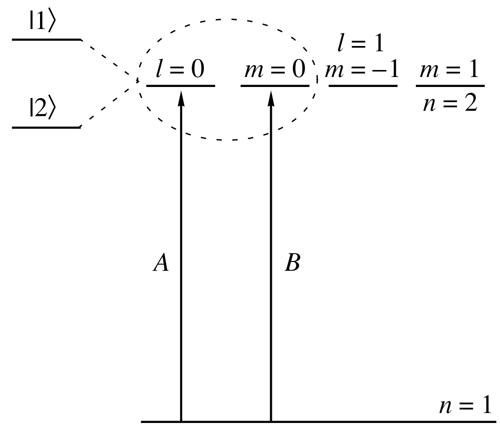

Fig. 3. Spinless model of the hydrogen atom. The information channel is formed from the input forbidden ($nlm \longrightarrow n'l'm'$) transition 100–200 and the output dipole-active transition 100–210.

the typical intraatom channel formed by two two-level systems constructed from two pairs of orthogonal states $A = \{|\psi_0\rangle, |\psi_1\rangle\}$ and $B = \{|\psi_0\rangle, |\psi_2\rangle\}$ of the same atom. As an example we shall use the spinless model of the hydrogen atom (Fig. 3): ψ_0 is the ground s state with $n = 1$, $\psi_{1,2}$ correspond to the s state with $l = 0$, $m = 0$ and the p state with $l = 1$, $m = 0$ of the first excited state $n = 2$.

In the absence of an external field the quantum channel does not transmit any coherent information, since the states $l = 0$, $m = 0$ and $l = 1$, $m = 0$ are not coupled. In the absence of an external electric field applied along the Z axis, the desired pair of four initially degenerate states with $n = 2$ splits as a result of the Stark effect and transforms into a pair of new eigenstates

$$|1\rangle = (|\psi_1\rangle + |\psi_2\rangle)/\sqrt{2}, \quad |2\rangle = (|\psi_1\rangle - |\psi_2\rangle)/\sqrt{2}$$

and the input state $l = 0$ oscillates with the frequency of the Stark shift:

$$|\psi_1(t)\rangle = \cos(\omega_s t)|\psi_1\rangle + \sin(\omega_s t)|\psi_2\rangle.$$

Therefore, on account of the applied electric field, the input states become entangled with the output states, which as a result contain coherent information about the input states.

In our model, Eq. (16) gives the operators $\hat{s}_{kl}$ in the form of a 3×3 matrices, where the third columns and rows correspond to a fictitious "vacuum" state $|0\rangle$:

$$\hat{s}_{11} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad \hat{s}_{12} = \begin{pmatrix} 0 & \sin(\omega_s t) & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{s}_{21} = \begin{pmatrix} 0 & 0 & 0 \\ \sin\omega_s t & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix},$$

$$\hat{s}_{22} = \begin{pmatrix} 0 & 0 & 0 \\ 0 & \sin^2(\omega_s t) & 0 \\ 0 & 0 & \cos^2(\omega_s t) \end{pmatrix}.$$

Zero values of the operators $\hat{s}_{12}$ and $\hat{s}_{21}$ correspond to the absence of coherent information at $t = 0$, i.e., the absence of entangled states. Choosing the input density matrix in the form $\hat{\rho}_{\text{in}} = \hat{I}/2$, we obtain the corresponding input–output density matrix:

$$\hat{\rho}_\alpha = \begin{pmatrix} \frac{1}{2} & 0 & 0 & \frac{x}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ \frac{x}{2} & 0 & 0 & \frac{x^2}{2} & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1-x^2}{2} \end{pmatrix},$$

where $x = \sin(\omega_s t)$ and the output density matrix $\hat{\rho}_{\text{out}}$ is diagonal with diagonal elements 1/2, $x^2/2$, and $(1 - x^2)/2$.

Calculating the nonzero eigenvalues $(1 \pm x^2)/2$ for $\hat{\rho}_\alpha$ and the entropies S_{out} and S_α, we obtain the coherent information

$$I_c = [(1 + x^2)\log_2(1 + x^2) - x^2\log_2(x^2)]/2.$$

This function is greater than zero everywhere with the exception of the point $x = 0$, where the coherent information is zero, and its maximum is one qubit with $x = \pm 1$, i.e., for the precession angle $\omega_s t = \pm\pi/2$. Hence it is evident that the coherent information about the states of the forbidden dipole transition is in principle accessible via the dipole transition using the Stark effect. Its average value in time is $\langle I_c \rangle = 0.46$ qubits.

The estimates made above indicate the potential possibility of observing experimentally the coherent effects due to the influence of the forbidden electronic transition on a dipole transition. Forbidden transitions were studied in [12, 13] as a potential source of information about the breakdown of spatial symmetry due to an interaction via weak neutral currents [14, 15]. If $I_c = 0$, then in principle only an incoherent effect of a forbidden transition via the population of the ground state n_0 is possible. In this case only one parameter—the population—can be measured, while the exact knowledge of the phase requires $I_c = 1$.

6. EXCHANGE OF COHERENT INFORMATION BETWEEN TWO CLOSED QUANTUM SYSTEMS

A variety of results associated with the exchange of coherent information between two atomic qubits, including analysis of the problem from the standpoint of the measure of quantum entanglements [16], analysis of the problem of eavesdropping [17], and a set of various experiments proposed in order to create a controllable entanglement in a system of two atoms [18, 19], has been published in the last few years. From the information standpoint coherent information exchanged in a system of two TLA coupled by a quantum channel depends on the specific form of the transformation realized by the quantum channel as well as on the initial states of the TLA. It is natural to take as the initial state the product of independent density matrices of the atoms: $\hat{\rho}_{1+2} = \hat{\rho}_{\text{in}} \otimes \hat{\rho}_2$.

In this section we give a systematic analysis of the processes leading to exchange of coherent information between two initially independent quantum systems. The following are included: (1) two unitarily coupled TLA (Section 6.1), (2) two TLA coupled by the quantum measurement procedure (Section 6.2), (3) an arbitrary system and its duplicated state (Section 6.3), (4) TLA and a photon field in free space (Section 6.4), and (5) two TLA interacting via the vacuum photon field (Section 6.5).

6.1. Two Unitarily Coupled Two-Level Atoms

We discuss first a noiseless deterministic quantum channel coupling two TLA (Fig. 1b). It can be described by a unitary two-atom transformation, given by the matrix elements $U_{ki,k'i'}$, $k, i, k', i' = 1, 2$. Then the superoperator of the transformation of channel $\mathscr{C}$, giving the mapping $\hat{\rho}_{\text{in}} \longrightarrow \hat{\rho}_{\text{out}} = \hat{\rho}'_2$, can be written in terms of the substitution symbol using the relation (6) with the operators

$$\hat{s}_{kl} = \sum_{\mu\nu} S_{kl,\mu\nu} |\mu\rangle\langle\nu|,$$

represented in accordance with Eqs. (8) and (12), by the matrix elements $\mathscr{C}$ in the form

$$S_{kl,\mu\nu} = \sum_{m\alpha\beta} \rho_{2\alpha\beta} U_{m\mu,k\alpha} U^*_{m\nu,l\beta}. \qquad (17)$$

The relation

$$\text{Tr}\hat{s}_{kl} = \sum_{\mu} S_{kl,\mu\mu} = \delta_{kl}$$

holds and gives the correct normalization, and the positiveness of the block matrix

$$(\hat{s}_{kl}) = \begin{pmatrix} \hat{s}_{11} & \hat{s}_{12} \\ \hat{s}_{21} & \hat{s}_{22} \end{pmatrix}$$

guarantees positiveness of $\mathscr{C}$.

For a transformation of the form $U = U_1 \otimes U_2$, which does not lead to the creation of entangled states, Eqs. (6) and (17) give $\mathscr{C} = \hat{\rho}'_2 \,\mathrm{Tr}\odot$, which signifies a transformation of the initial state $\hat{\rho}'_1$ of the first TLA into the final state, which is not entangled with the state $\hat{\rho}'_2 = U_2 \hat{\rho}_2 U_2^+$ of the second TLA.

Relation (17) can be simplified by considering pure states $\hat{\rho}'_2$, so that in combination with the possibility of choosing an arbitrary transformation U without entanglement it is useful, specifically, to single out especially the case of the state $\rho_{2\alpha\beta} = \delta_{\alpha\beta}\delta_{\alpha\alpha_0}$. Taking account of the linearity of the dependence of $S_{kl,\,\mu\nu}$ on $\hat{\rho}'_2$ and the convexity of the coherent information I_c as a function of $\mathscr{C}$ [20], the analysis of Eq. (17) can be reduced to analysis of the relation

$$S_{kl,\,\mu\nu} = \sum_m U_{m\mu,\,k\alpha_0} U^*_{m\nu,\,l\alpha_0}, \qquad (18)$$

which means that the quantum channel is described only by a unitary transformation U. Here the summation extends only over the states $|m\rangle$ of the first atom after the entangling transformation.

The coherent information transmitted, in the present system of two coupled TLA with

$$\hat{\rho}_{\mathrm{in}} = \hat{I}/2, \quad (\hat{\rho}_2)_{12} = \sqrt{(\hat{\rho}_2)_{11}[1-(\hat{\rho}_2)_{11}]}$$

is shown in Fig. 4. It is a convex function of $\hat{\rho}_2$ with a maximum at the boundary, $\rho_{11} = 0.1$. Just as in the case of one TLA, the coherent information preserves the typical threshold character of the dependence on the coupling angle, which describes the degree of coherent coupling of two TLA with respect to independent fluctuations of the second TLA.

6.2. Two Two-Level Atoms Coupled by the Quantum Measurement Procedure

We shall consider a special type of quantum channel coupling two TLA,[4] which can be described by a superoperator $\mathscr{C}$, defining the quantum measurement procedure. This procedure is related with a different approach to defining the quantum information [21], based on the so-called measured information.

Let us consider first a channel consisting of two identical two-level systems. In terms of the wave function the corresponding transformation of the complete measurement of the state of the first TLA has the form

$$\psi \otimes \varphi \longrightarrow \sum a_i |\phi_i\rangle|\phi_i\rangle, \quad a_i = \langle\phi_i|\psi\rangle. \qquad (19)$$

[4] In reality, the results of the present section are valid not only for two TLA but also for any quantum systems with finite dimension.

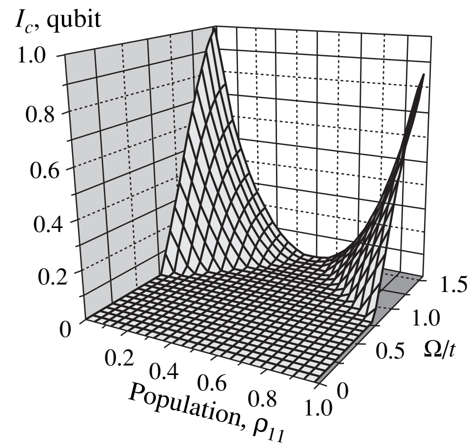

Fig. 4. Coherent information transmitted between two TLA coupled by a unitary transformation, as a function of the matrix element ρ_{11} of the diagonal initial density matrix of the second TLA and the procession angle $\varphi = \Omega t$.

This transformation describes the entanglement of certain basis states $|\phi_i\rangle$, which does not depend on the initial state φ of the second TLA. The latter is an indicator of the measuring setup, preserving completeness of information on the basis states in the initial state $\psi = \sum a_i|\phi_i\rangle$. Relation (19), given in the form of a single-valued transformation of the wave function, in reality is not a linear transformation with respect to φ and therefore cannot represent a deterministic transformation, not being unitary. The corresponding representation in terms of the two-atom density matrices has the form

$$\hat{\rho}_{12} \longrightarrow \sum_i \sum_j \langle\phi_i|\langle\phi_j|\hat{\rho}_{12}|\phi_j\rangle|\phi_i\rangle|\phi_i\rangle|\phi_i\rangle\langle\phi_i|\langle\phi_i|. \qquad (20)$$

It is linear with respect to $\hat{\rho}_{12}$ and satisfies the standard conditions of physical realizability [7, 20], i.e., completely positive and preserves normalization. The density matrix has the form $\sum p_i|\phi_i\rangle|\phi_i\rangle\langle\phi_i|\langle\phi_i|$, so that $S(\hat{\rho}_{12}) = S(\hat{\rho}_2)$, and in accordance with the relations of Section 3 the one-time coherent information is zero. This is due to the classical nature of the information, represented here only by the classical indices i.

The quantum transformation superoperator coupling two TLA in the case of a two-time channel can be obtained from the relation (6) with $\hat{s}_{kl} = |\phi_k\rangle\langle\phi_k|\delta_{kl}$, $\langle k| \longrightarrow \langle\phi_k|$ and $|k\rangle \longrightarrow |\phi_k\rangle$, which after taking the trace with respect to the first TLA and replacing $\hat{\rho}_{12}$ by the substitution symbol $\odot$ becomes

$$\mathscr{M} = \sum_k \hat{P}_k \mathrm{Tr}_1 \hat{E}_k \odot, \qquad (21)$$

where $\hat{P}_k = |\phi_k\rangle\langle\phi_k|$ are orthogonal projectors, representing the eigenstates of the “indicator” variable of the second TLA, and $\hat{E}_k = |\phi_k\rangle\langle\phi_k|$ describes the orthogonal expansion of unity. It is constructed from the same operators, giving here the transformation of the quantum-classical reduction $\mathrm{Tr}_1 \hat{E}_k \odot = \langle\phi_k| \odot |\phi_k\rangle$, which gives the procedure for obtaining the classical information k from the first system. Applying the transformation (21) to $\hat{\rho}_{\mathrm{in}}$ and using Eq. (9), we obtain for the corresponding output and input–output density matrices

$$\hat{\rho}_{\mathrm{out}} = \sum_k \tilde{p}_k |\phi_k\rangle\langle\phi_k|,$$
$$\hat{\rho}_{\alpha} = \sum_k \tilde{p}_k |\pi_k\rangle|\phi_k\rangle\langle\phi_k|\langle\pi_k|, \tag{22}$$

where

$$\tilde{p}_k = \langle\phi_k|\hat{\rho}_{in}|\phi_k\rangle = \sum_i p_i |\langle\phi_k|i\rangle|^2$$

are the eigenvalues of the probabilities for the reduced density matrix, and

$$|\pi_k\rangle = \sum_i \sqrt{p_i/\tilde{p}_k}\langle\phi_k|i\rangle|i\rangle$$

are the normalized modified input states, coupled after the quantum measurement procedure with the output states $|\phi_k\rangle$. It should be noted especially that, as follows from Eqs. (22), there is no exchange of coherent information in the system, since the vectors $|\phi_k\rangle$ are orthogonal and the entropies of the density matrices (22) are obviously the same. Conversely, the measured information, introduced in [21], in this case is different from zero.

There is no difficulty in extending this result to the case of a channel of a more general form, where the output system has a structure that is different from the initial system and is described by a different Hilbert space. The latter difference is taken into account by replacing in the preceding relations the basal states $|\phi_i\rangle$ of the second system by a different orthogonal set $|\varphi_i\rangle = V|\phi_i\rangle$, where V is the isometric transformation from the Hilbert space of the states H_1 of the first system into a different Hilbert space H_2 of the second system. After simple, obvious transformations, we obtain the same final result—the absence of coherent information transmitted in such a channel. This result is characteristic for coherent information, in contrast to other information approaches (see, e.g., [21]).

It is of interest to discuss quantum-measurement transformations of a more general type, specifically, the procedure of indirect (generalized) measurement, first introduced in application to problems of the theory of optimal quantum detection and measurement in [22], and in a more general form of the nonorthogonal expansion of unity $\hat{\mathscr{E}}(d\lambda)$ in [23] ($\hat{\mathscr{E}}(d\lambda)$ is the equivalent of a positive-definite operator-valued measure (POVM), used in semiclassical variants of quantum information theory and quantum theory of optimal detection/measurement [5, 24, 25]). The corresponding transformation of the indirect measurement is obtained by averaging the transformation of the direct measurement, applied not directly to the system of interest but rather to its combination with an arbitrary independent system. In the general case the indirect measurement superoperator can be represented as

$$\mathcal{M} = \sum_q \hat{P}_q \mathrm{Tr}\hat{\mathscr{E}}_q \odot, \tag{23}$$

where $\hat{P}_q$ describe an arbitrary set of orthogonal projectors, and $\hat{\mathscr{E}}_q$ is a general nonorthogonal expansion of unity in the space H (POVM). We note that $\hat{\mathscr{E}}_q = |\varphi_q\rangle\langle\varphi_q|$ describes the case of a "pure" POVM, first introduced precisely in this form in the quantum theory of detection/measurement [22]. It corresponds to a complete measurement in $H \otimes H_a$ with the choice of the singular density matrix for the initial state of an auxiliary independent system $\rho^a_{bc} = \delta_{b0}\delta_{bc}$.

In Eq. (23) the classical index q represents the information exchange between the initial state and final state of the output. Since the number N_q of values of q can be greater than the dimension $\dim H$, it can be inferred that a definite amount of coherent information might be attained as a result of this excess. The corresponding output and input–output density matrices have the form

$$\hat{\rho}_{out} = \sum_q \tilde{p}_q \hat{P}_q,$$
$$\hat{\rho}_{\alpha} = \sum_{qij} \sqrt{p_i p_j}\langle j|\hat{\mathscr{E}}_q|i\rangle \hat{P}_q \otimes |i\rangle\langle j|, \tag{24}$$

where $\tilde{p}_q = \mathrm{Tr}\hat{\mathscr{E}}_q \hat{\rho}_{in}$ describe the probabilities of states determined by indirect measurement. For the case of complete indirect measurement, based on the quantum analog [2] of the classical theorem of no increase in information in successive transformations of data and on the above-proved result concerning the complete direct measurement, it is not difficult to substantiate theoretically and confirm by numerical calculations that it is impossible to obtain coherent information. Therefore, to obtain as a result of a measurement procedure a nonzero amount of coherent information it is necessary to use incomplete ("soft") quantum measurements which require an independent, more detailed investigation.

6.3. Quantum Duplication Procedure

In counterpoint to the above-studied dequantizing-type measurement procedure, determined by the transformation (20), which completely destroys coherent information, here we shall examine a transformation in a channel, shown in Fig. 1c, which preserves the coherent information:

$$\hat{\rho}_{12} \longrightarrow \hat{\rho}'_{12} = \sum_{ij} \langle\phi_i|\mathrm{Tr}_2\hat{\rho}_{12}|\phi_j\rangle|\phi_i\rangle|\phi_i\rangle\langle\phi_j|\langle\phi_j|.$$

It does not ignore the phase relations between the various ϕ_i because of the use of the off-diagonal matrix elements of the input density matrix $\hat{\rho}_1 = \hat{\rho}_{\mathrm{in}}$.

For the initial density matrix in the form of a product $\hat{\rho}_{\mathrm{in}} \otimes \hat{\rho}_2$, in terms of the transformation $\hat{\rho}_{\mathrm{in}} \longrightarrow \hat{\rho}'_{12}$ from H to $H \otimes H$ the corresponding superoperator has the form

$$\mathfrak{Q} = \sum_{ij} |\phi_i\rangle|\phi_i\rangle\langle\phi_j|\langle\phi_j|\langle\phi_i| \odot |\phi_j\rangle. \qquad (25)$$

This superoperator determines the transformation of a coherent measurement in counterpoint to an incoherent measurement, studied in [21]. The transformation of the coherent measurement converts $\hat{\rho}_{\mathrm{in}}$ into an $\hat{\rho}_2$-independent state

$$\hat{\rho}_{\mathrm{out}} = \hat{\rho}'_{12} = \sum_{ij} \langle\phi_i|\hat{\rho}_{in}|\phi_j\rangle|\phi_i\rangle|\phi_i\rangle\langle\phi_j|\langle\phi_j|, \qquad (26)$$

which results in duplication of the eigenstates ϕ_i of the input by the same states of the indicator variable

$$\hat{k} = \sum_k k|\phi_k\rangle\langle\phi_k|.$$

The pure input states transform into pure states of the composite (1 + 2) system by means of duplication of the indicator states:

$$\psi \longrightarrow \sum_i \langle\phi_i|\psi\rangle|\phi_i\rangle|\phi_i\rangle,$$

which repeats the mapping (19), which gives a multivalued description of the corresponding superoperator transformation. Of course, only the input states equal to the eigenstates ϕ_k of the chosen indicator variable are duplicated without distortion as a result of the incompatibility of the nonorthogonal states; this is the basic theorem of the impossibility of quantum cloning [26]. The entropy of the output state with density matrix (26), possessing the same matrix elements as $\hat{\rho}_{\mathrm{in}}$, obviously is identical to the entropy of the input state, $S_{\mathrm{out}} = S_{\mathrm{in}} = S[\hat{\rho}_{\mathrm{in}}]$, on account of the conservation of the coherence of all pure input states.

For combined input–output states the transformation (25) leads to the density matrix (9) in the space $H \otimes H \otimes H$ of the form

$$\hat{\rho}_\alpha = \sum_{kl} |\phi_k\rangle|\phi_k\rangle\langle\phi_l|\langle\phi_l| \otimes \sqrt{\tilde{p}_k\tilde{p}_l}|\chi_k\rangle\langle\chi_l|, \qquad (27)$$

where $\tilde{p}_k$ and $|\chi_k\rangle$ are the same as above; this makes it possible to construct a spectral expansion of the density matrix in the form

$$\hat{\rho}_{\mathrm{in}} = \sum_k \tilde{p}_k|\chi_k\rangle\langle\chi_k|.$$

Keeping in mind the fact that the first term of the tensor product in Eq. (27) is a set of transition projection operators $\hat{P}_{kl}$, $\hat{P}_{kl}\hat{P}_{mn} = \delta_{lm}\hat{P}_{kn}$, it is easy to prove the algebraic rule used for an arbitrary scalar function f:

$$f\left(\sum_{kl}\hat{P}_{kl} \otimes \hat{R}_{kl}\right) = \sum_{kl}\hat{P}_{kl} \otimes f(\hat{R})_{kl},$$

where $\hat{R} = (\hat{R}_{kl})$ is a block matrix, and

$$\mathrm{Tr}f\left(\sum_{kl}\hat{P}_{kl} \otimes \hat{R}_{kl}\right) = \mathrm{Tr}f(\hat{R}).$$

Here $\hat{R} = (\sqrt{\tilde{p}_k\tilde{p}_l}|\chi_k\rangle\langle\chi_l|)$, which equals simply $\|\chi\rangle\rangle\langle\langle\chi\|^+$ with $\|\chi\rangle\rangle_{ki} = \sqrt{\tilde{p}_k}\chi_{ki}$, since this corresponds to a vector in the space $H \otimes H$. The eigenvalues λ_k of this matrix are {1, 0, 0, 0} with a single nonzero eigenvalue, corresponding to the vector $\|\chi\rangle\rangle$.

A calculation of the exchange entropy gives $S_{\mathrm{e}} = 0$ and therefore $I_c = S_{\mathrm{in}}$. This means that the quantum duplication procedure does not decrease the volume of coherent information in the channel $1 \longrightarrow (1 + 2)$ irrespective of whether or not the indicator $\hat{k}$ is compatible with the input density matrix, i.e., $[\hat{k}, \hat{\rho}_{\mathrm{in}}] = 0$.

If the channel considered is reduced to the channel shown in Fig. 1b and studied in the preceding section by taking the trace over the first or second system in Eq. (26), we obviously arrive at the measuring procedure examined in Section 6.2. As a result, we can conclude that coherent information is not associated with each system separately, i.e., it is strongly coupled with both systems. The specific nature of the quantum information, studied above, can be used, specifically, in algorithms for correcting quantum errors [27] or for producing stable entangled states [28].

6.4. Two Level Atom–Vacuum Field Channel

We now consider the interaction between a TLA and a vacuum electromagnetic field, i.e., the process of electromagnetic emission, as an information channel (Fig. 1b), which compared with a TLA in a given laser field (see Section 4) introduces a new object—the photon vacuum field—as the output.

For this purpose we shall employ a reduced model of the field based on the reduction of the Hilbert space in the Fock representation (Fig. 5). In a more general

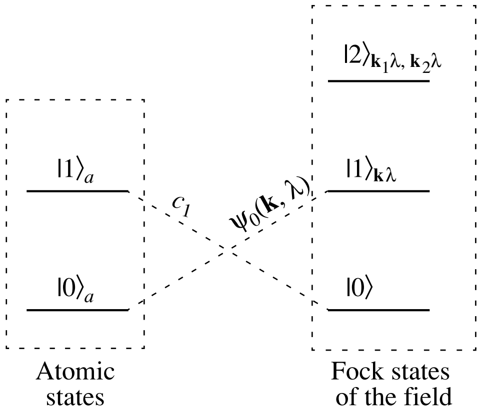

Fig. 5. Structure of the compound Hilbert space of the atom–field system. For the vacuum initial state of the field both atomic states and only two Fock states of the field ($|0\rangle$ and $|1\rangle$) are included in the dynamics of the atom–field composite system, which can be defined by only two states $|0\rangle_a|1\rangle_{\mathbf{k}\lambda}$ and $|1\rangle_a|0\rangle$, described by time-dependent functions $\psi_0(\mathbf{k}, \lambda)$ and c_1, respectively.

terminology, this problem corresponds to the problem of the dynamics of the interaction of a two-level system with a multimodal system of linear oscillators [29]. The solution of the latter problem on long-time scales, to which we shall confine our attention here, corresponds to the standard description of the emission of a single photon. Moreover, for purposes of information analysis of a system consisting of an atom and a field, there is no need to describe the coherent dependence of the wave function $\psi_0(\mathbf{k}, \lambda)$ of the photon on the photon wave vector (and polarization), since only the total probability of emission of the photon is important.

In the basis of states of the free atom and the Fock states of the free field for a vacuum initial state $\alpha_0 = 0$ we obtain from the relation (18)

$$S_{kl,\,\mu\nu} = \sum_m U_{m\mu,\,k0} U^*_{m\nu,\,l0},$$

where the Greek indices are used to denote states of the photon field, which in general correspond to the number of photons and their spatial coordinates or wave vectors. The calculation of this superoperator, performed on the basis of a unitary matrix of the temporal evolution of the atom–field system with matrix elements $U_{m\mu,\,k0}$, is illustrated in Tables 1 and 2.

Choosing $\psi_0(\mathbf{k}, \lambda)$ as the basis element of a single-photon subspace of states of the field reduces the matrix $S_{kl,\,\mu\nu}$ of the superoperator to a nonoperator transformation matrix, which in terms of the matrices $\hat{s}_{kl}$ has the form

$$\hat{s}_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix},\quad \hat{s}_{12} = \begin{pmatrix} 0 & (1-e^{-\gamma t})^{1/2} \\ 0 & 0 \end{pmatrix},\quad \hat{s}_{21} = \begin{pmatrix} 0 & 0 \\ (1-e^{-\gamma t})^{1/2} & 0 \end{pmatrix},\quad \hat{s}_{22} = \begin{pmatrix} e^{-\gamma t} & 0 \\ 0 & 1-e^{-\gamma t} \end{pmatrix}, \tag{28}$$

where $|c_1|^2 = \exp(-\gamma t)$ describes the decay of the population of an initially completely populated excited state of the atom, and

$$\int \sum |\psi_0(\mathbf{k}, \lambda)|^2 d\mathbf{k} = 1 - \exp(-\gamma t)$$

describes the total probability of detecting a photon. It follows from Eqs. (28) that the structure of the photon is of no significance, and the transmitted information is determined only by the probability of emission of a photon by the time t. This reduction of the photon field (only the photon numbers μ, ν = 0, 1 are important) reduces it to an equivalent two-level system.

Applying the transformation (28) to the input density matrix

$$\hat{\rho}_{in} = \begin{pmatrix} \rho_{11} & \rho_{12} \\ \rho_{12} & 1-\rho_{11} \end{pmatrix},$$

where we have confined ourselves to the case of purely real off-diagonal matrix elements, we obtain the output density matrix

$$\hat{\rho}_{out} = \begin{pmatrix} \rho_{11} + \rho_{22} e^{-\gamma t} & \rho_{12}(1-e^{-\gamma t})^{1/2} \\ \rho_{12}(1-e^{-\gamma t})^{1/2} & \rho_{22}(1-e^{-\gamma t}) \end{pmatrix},$$

and the corresponding input–output density matrix, which for the case $\rho_{12} = 0$ has the form

$$\hat{\rho}_\alpha = \left(\begin{array}{cc|cc} \rho_{11} & 0 & 0 & [\rho_{11}\rho_{22}(1-e^{-\gamma t})]^{1/2} \\ 0 & \rho_{22}e^{-\gamma t} & 0 & 0 \\ \hline 0 & 0 & 0 & 0 \\ [\rho_{11}\rho_{22}(1-e^{-\gamma t})]^{1/2} & 0 & 0 & \rho_{22}(1-e^{-\gamma t}) \end{array}\right).$$

For $t \longrightarrow \infty$ this expression gives a completely entangled state (in the sense of the absence of classical correlations, since it is a pure state) of the atom–photon system, leading to transfer to the photon of coherent fluctuations of the atomic state, which are equivalent to a mixed ensemble. The corresponding eigenvalues are

$$\lambda_\alpha = \{0, 0, 1 - \rho_{22}\exp(-\gamma t), \rho_{22}\exp(-\gamma t)\}.$$

The nonzero values describe the probabilities of atomic states at time t. The eigenvalues for the output density matrix (photon + vacuum) $\hat{\rho}_{\text{out}}$ are

$$\lambda_{\text{out}} = \{\rho_{22}[1 - \exp(-\gamma t)], 1 - \rho_{22}[1 - \exp(-\gamma t)]\}$$

and describe the probability of observing whether the photon is emitted or not. The set of quantities presented above determines the characteristic values of the probabilities of the combined input–output density matrix and the partial density matrices. The coherent information given by the corresponding entropy difference assumes the form

$$\begin{aligned} I_c &= x\rho_{22}\log_2(x\rho_{22}) \\ &- (1-\rho_{22}+x\rho_{22})\log_2[1-(1-x)\rho_{22}] \\ &+ (1-x\rho_{22})\log_2(1-x\rho_{22}) \\ &- (1-x)\rho_{22}\log_2(\rho_{22}-x\rho_{22}), \end{aligned} \tag{29}$$

where $x = \exp(-\gamma t)$. This formula is applicable for $I_c > 0$, while in the opposite case $I_c = 0$. The corresponding critical moment in time is determined by the relation $\exp(-\gamma t) = 1/2$, which corresponds to the probability $1 - \rho_{22}[1 - \exp(-\gamma t)]$ of the absence of a photon being equal to the probability $1 - \rho_{22}\exp(-\gamma t)$ of the bottom atomic level being occupied.

The results of the calculation of the coherent information are presented in Fig. 6 for two special cases: $\rho_{12} = 0$ (Fig. 6a) and $\rho_{11} = 1/2$, $0 \le \rho_{12} \le 1/2$ (Fig. 6b). It is easy to see from Fig. 6a that the coherent information is symmetric with respect to the symmetry point $\rho_{11} = 1/2$. A further increase of the population of the excited state $\rho_{22} = 1 - \rho_{11}$ and the corresponding level of photon emission does not increase the amount of coherent information. This is due to the decrease in the input entropy, which determines the potential maximum of coherent information. For the same reasons, the coherent information decreases if a nonzero coherent correction is made to the initial density matrix of the atom in the form of a state with maximum entropy and vanishes for a purely coherent initial state (Fig. 6b).

In accordance with Section 3 and taking account of the fact that the initial state of the field is pure, the one-time information is equal to the entropy difference only for the photon field represented by the density matrix $\hat{\rho}_{\text{out}}$ and the initial atomic state represented by $\hat{\rho}_{\text{in}}$. For a pure initial state of the atom in the form of an excited state $|2\rangle$ we obtain for all times the nonzero coherent information

$$I_c = -x\log_2 x - (1-x)\log_2(1-x),$$

which gives one qubit for the time when $x = 1/2$ and the population of the excited state coincides with the probability of emission of a photon.

Table 1. Unitary transformation atom–field–atom–field $U_{m\mu,\,k\alpha}$ for a vacuum initial state of the photon field; the indices m and k enumerate the atomic photons, μ and α are the photon numbers

$k\alpha$	$m\mu$			
	00	01	10	11
00	1	0	0	0
01	–	–	–	–
10	0	$\psi_0(\mathbf{k}, \lambda)$	c_1	0
11	–	–	–	–

Note: The second and fourth rows are the matrix elements that do not appear in the matrix elements $S_{kl,\,\mu\nu}$ which are computed (see Table 2).

Table 2. Atom–field superoperator $S_{kl,\,\mu\nu}$, setting the transformation $|k\rangle\langle l| \longrightarrow |\mu\rangle\langle\nu|$. The indices k and l enumerate the atomic photons, and μ, ν are the photon numbers

kl	$\mu\nu$			
	00	01	10	11
00	1	0	0	0
01	0	0	$\psi_0(\mathbf{k}, \lambda)$	0
10	0	$\psi_0^+(\mathbf{k}, \lambda)$	0	0
11	$\|c_1\|^2$	0	0	$\psi_0(\mathbf{k}, \lambda) \times \psi_0^+(\mathbf{k}', \lambda')$

6.5. *Atom-to-Atom Transmission of Coherent Information Via a Free Vacuum Field*

Let us consider a quantum channel of the type $1 \longrightarrow 2$ (Fig. 1b), where information is transferred from one atom to another through free space by emission of a photon, assuming that initially the second atom is the ground state. In addition, we introduce a limitation on the scale of the times considered, excluding from our analysis fast processes occurring on times of the order of and less than the period of atomic oscillations, i.e., ignoring the discrete nature of the electromagnetic signal that is associated with interatomic retardation [30–33]. In this approximation the problem under consideration is a Dicke problem [34], for which the solution in terms of two time-decaying symmetric and antisym-

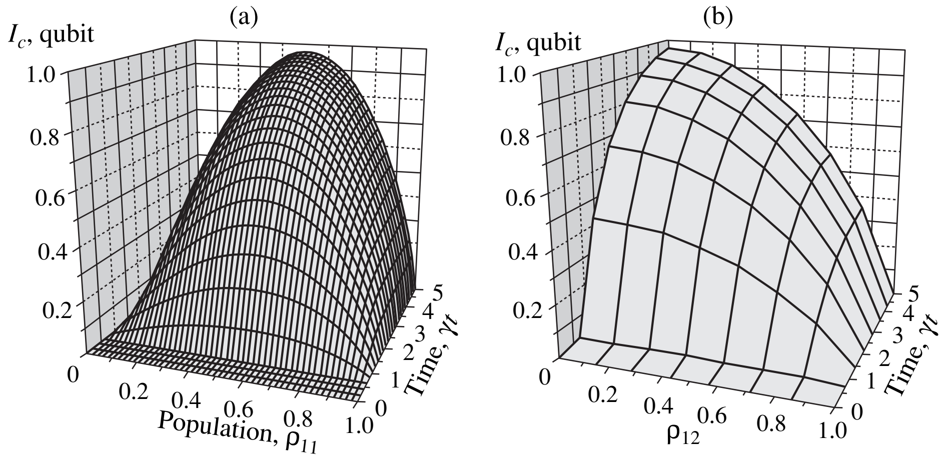

Fig. 6. Coherent information transmitted by the atom–field quantum channel as a function of time and the output density matrix of the atom: (a) the density matrix is diagonal with matrix element ρ_{11} of the ground state; (b) the density matrix is described by the sum of $\hat{I}/2$ and the real ("cosine"-type) coherent addition in the form of the off-diagonal term $\rho_{12}\hat{\sigma}_1$.

metric Dicke states $\|s\rangle\rangle = (|1\rangle|2\rangle + |2\rangle|1\rangle)/\sqrt{2}$, $\|a\rangle\rangle = (|1\rangle|2\rangle - |2\rangle|1\rangle)/\sqrt{2}$ and a stable vacuum state $\|0\rangle\rangle = |1\rangle|1\rangle$ in the following form is well known:

$$\begin{gathered} c_s(t) = c_s(0)\exp[-(\gamma_s/2 + i\Lambda)t], \\ c_a(t) = c_a(0)\exp[-(\gamma_a/2 + i\Lambda)t], \\ c_0(t) = c_0(0) \\ + [c_s(0)^2 + c_a(0)^2 - c_s(t)^2 - c_a(t)^2]^{1/2} e^{i\xi(t)}, \end{gathered} \tag{30}$$

where $c_0(t)$ is the complex amplitude of the vacuum component $|1\rangle|1\rangle$, including the incoherent correction due to the spontaneous radiative transitions from excited atomic states, $\xi(t)$ is the uniformly distributed phase of atomic oscillations, $\gamma_{s,a}$ and Λ are the decay rates and the frequency splitting (frequency shift), respectively, and $c_{s,a}$ are the complex amplitudes of the Dicke states.

In terms of multiplicative combinations of individual atomic states $|i\rangle|j\rangle$ for the corresponding amplitudes of the initial states $c_{12}(0) = 0$ and $c_{22}(0) = 0$ the dynamics of the system under study is described in accordance with the dynamics of Dicke states, determined by the relations (30), for the following equations:

$$\begin{gathered} c_{11}(t) = c_{11}(0) + f(t)e^{i\xi(t)}c_{21}(0), \\ c_{21}(t) = f_s(t)c_{21}(0), \quad c_{12}(t) = f_a(t)c_{12}(0), \\ c_{22}(t) = 0, \\ f(t) = \{1 - [\exp(-\gamma_s t) + \exp(-\gamma_a t)]/2\}^{1/2}, \\ f_s(t) = \{\exp[-(\gamma_s/2 + i\Lambda)t] \\ + \exp[-(\gamma_a/2 - i\Lambda)t]\}/2, \\ f_a(t) = \{\exp[-(\gamma_s/2 + i\Lambda)t] \\ - \exp[-(\gamma_a/2 - i\Lambda)t]\}/2. \end{gathered}$$

Using these expressions for the input operators of the form $c_{k1}(0)c^*_{l1}(0)|k\rangle\langle l|$ for the first atom and then averaging them over the final states of the first atom and fluctuations of the atomic field (the latter are represented here only by the variable $\xi(t)$), we obtain the symbolic representation of the superoperator of the transformation of the channel $\hat{\rho}^{(1)}(0) \longrightarrow \hat{\rho}^{(2)}(t) = \mathscr{C}(t)\hat{\rho}^{(1)}(0)$ and the corresponding operators $\hat{s}_{kl}$ in the form

$$\begin{gathered} \mathscr{C}(t) = |1\rangle\langle 1| \odot |1\rangle\langle 1| + [f(t)^2 + |f_s(t)|^2]|1\rangle \\ \times \langle 2| \odot |2\rangle\langle 1| + |f_a(t)|^2|2\rangle\langle 2| \odot |2\rangle\langle 2| \\ + f_a(t)|2\rangle\langle 2| \odot |1\rangle\langle 1| + f^*_a(t)|1\rangle\langle 1| \odot |2\rangle\langle 2|, \\ \hat{s}_{11} = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}, \quad \hat{s}_{12} = \begin{pmatrix} 0 & f^*_a(t) \\ 0 & 0 \end{pmatrix}, \\ \hat{s}_{21} = \begin{pmatrix} 0 & 0 \\ f_a(t) & 0 \end{pmatrix}, \\ \hat{s}_{22} = \begin{pmatrix} f(t)^2 + |f_s(t)|^2 & 0 \\ 0 & |f_a(t)|^2 \end{pmatrix}. \end{gathered} \tag{31}$$

To make the problem more concrete we shall consider two identical atoms with parallel dipole moments, directed perpendicular to the vector connecting the atoms under study. Then only two dimensionless variables are important: γt, where γ describes the radiative

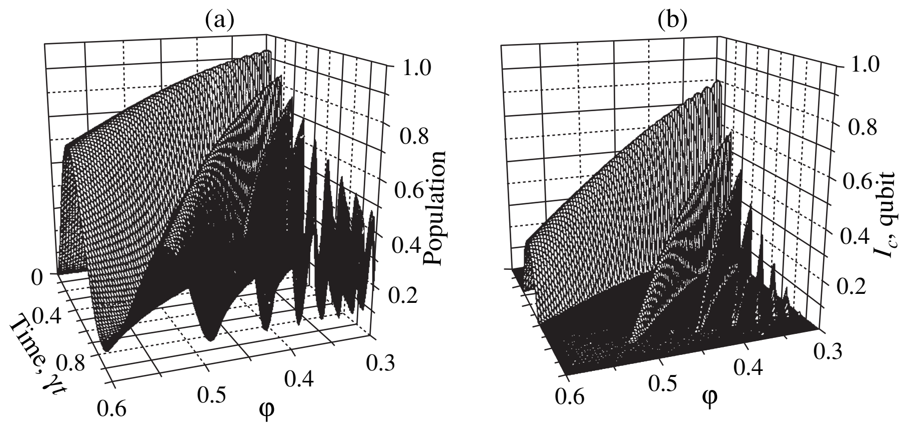

Fig. 7. (a) Population of the excited state of the second atom and (b) the coherent information in a system of two atoms interacting through free space as a function of the dimensionless time γt and the interatomic distance $\varphi = \omega_0 R/c$. The input density matrix corresponds to a state with maximum entropy $\hat{\rho}_{in} = \hat{I}/2$.

decay rate of an isolated atom, and a dimensionless interatomic distance $\varphi = k_0R$, where R is the distance between the atoms and k_0 is the modulus of the wave vector corresponding to the frequency of the atom. The dimensionless two-atom radiative decay rates and the frequency shift due to the short-range dipole-dipole interaction are described by the corresponding relations [19, 28–33]:

$$\gamma_{s,a}/\gamma = 1 \pm g, \quad \Lambda/\gamma = (3/4)/\varphi^3,$$

where $g = (3/2)(\varphi^{-1}\sin\varphi + \varphi^{-2}\cos\varphi - \varphi^{-3}\sin\varphi)$.

The corresponding coherent information can be calculated as done in Section 6.4. Using the correspondence $\exp(-\gamma t) \longrightarrow f(t)^2 + |f_s(t)|^2$, the operators $\hat{s}_{kl}$ for the two cases are completely similar and the coherent information, once again, is described by the same relation (29) with $x = f(t)^2 + |f_s(t)|^2$. Nonetheless, in this case the dependence considered, as compared with the case of one atom (see Section 6.4), has specific qualitative features on account of the oscillatory character of the function $|f_{s,a}(t)|^2$ as a function of the interatomic distance φ.

If there were no oscillations due to the quasielectrostatic short-range dipole-dipole interaction, i.e., if one could set $\Lambda = 0$, then the coherent information would always be zero, since the threshold $x < 0.5$ cannot be reached. The parameter $1 - x$ corresponds to the population of the excited state of the second atom with the initial state $|2\rangle$ of the first atom, and for the optimal, from the standpoint of information, value of the population of the first atom ρ_{22}, equal to half its initial population, we obtain $1 - x \le 1/4$ and correspondingly $x \ge 3/4$. The oscillations in $|f_a(t)|^2$ lead to interference between two decaying Dicke components, so that the maximum of the population $n_2 = 1 - x$ also reaches larger values right up to $n_2 = 1$, and the corresponding coherent information becomes different from zero.

The functions $n_2(\varphi, \gamma t)$ and $I_c(\varphi, \gamma t)$, calculated using the relation (29), are shown in Fig. 7. They serve as a universal measure for a system of two atoms, being independent of their frequency or the magnitude of the dipole moment (the latter is valid only for a fixed geometry of the system, described above).

As one can see in Fig. 7a the population decays rapidly as a function of time because of the rapid decay of the short-lived Dicke component. The population and the coherent information undergoes strong oscillations (Fig. 7b) for small interatomic distances φ. As $\varphi \longrightarrow 0$ the population of the long-lived Dicke state remains substantial for unlimited long times, but no coherent information is associated with it because the other component decays completely.

7. CONCLUSIONS

It was shown in this work that the concept of coherent information can be used for obtaining the most general description of the interaction between two real quantum systems, including systems of qualitatively different physical nature, and for determining the role of quantum coherence in the composite system.

It was shown for a TLA in a resonant laser field that coherent information in the system does not increase with increasing intensity of the applied field, provided that the relaxation processes themselves are not suppressed.

The hydrogen atom was considered as an example of the information exchange between subsystems of a single system. It was shown that under the action of an applied electric field coherent information exchange occurs between forbidden and dipole-active atomic transitions as a result of the interaction due to the Stark effect.

It was shown for two unitarily coupled TLA that the maximum possible value of the coherent information $I_c = 1$ qubit is reached for maximum entanglement and

$I_c = 0$ for any type of measurement procedures studied in Section 6.2.

It was shown for information exchange between TLA and a free photon field in the process of emission of electromagnetic radiation that the coherent information reaches the threshold of nonzero values at the critical point of the decay exponential $\exp(-\gamma t) = 1/2$, where the probability of there being no emitted photon is equal to the population of the lower atomic state. At the maximum the coherent information can reach the value $I_c = 1$ qubit.

It was shown for information exchange between two atoms by means of the vacuum field, when the atoms are separated by a distance of the order of the wavelength, that the coherent information is nonzero only as a result of coherent oscillations between the Dicke states, which are due to short-range dipole-dipole quasielectrostatic interaction with spatial dependence $\propto 1/R^3$. In contradistinction to this, the semiclassical information extracted using the quantum detection procedure is associated with population correlations [28].

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