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Generalized Quantum Measurements of Continuous Variables

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Abstract—The general concept of generalized quantum measurements with minimal distortion of initial information, which was introduced in our earlier publication [1] is applied to the problem of continuous quantum variables measurement. The qualitative features of such measurements are revealed and compared with a standard completely dequantizing measurement, which is described by the corresponding positive-definite operator-valued measure.

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1. INTRODUCTION

In contrast to classical systems, quantum physical systems possess a specific feature manifested in a much larger variety of their properties depending on the method of their application. Theoretically, this is reflected in the corresponding variety of aspects in which an apparently the same (in the classical limit) problem is considered. Therefore, it is not surprising that it necessary to return from time to time to analysis of the same problems from a different angle of view. Experience (in particular, gained in discussion of the Einstein–Podolsky–Rosen paradox [2]) shows that such a revision may have a practical yield (in the example considered here, this is the development of quantum cryptography [3]).

Quantum measurement is one of fundamental problem in quantum theory; in a discussion of this problem, a large number of different aspects can be traced [4–7]. In the contents of our study, we can single out the following two aspects pertaining to the possibility of displaying information contained in quantum systems by classical systems. We are speaking, first, of the theoretical possibility of obtaining absolutely precise results of measurements for jointly measurable quantities [8] and second, of the possibility of obtaining approximate results (correct to within the minimal quantum indeterminacy) of measurement for jointly immeasurable quantum variables [9] (see [10] for the discussion of this problem using a more general mathematical technique and for the review of the literature).

The assumption concerning the classical nature of the measuring system appears as quite natural at the stage of mastering the foundations of quantum theory, at which experimental intrusion in a quantum object for extracting information from it appeared as a rough process. However, this limitation should be revised, in our opinion, at present, when series of reversible quantum-mechanical transformations are being freely carried out in quantum systems of various types and quantum systems such as photons are frequently used in the protocols of quantum cryptography. Namely, in the general case, we should identify a meter (subsequently referred to just as meter) with an essentially quantum object, which gives the result of measurement in the quantum-mechanical or classical form depending on its surroundings; this must be reflected in the specific form of the transformation which is actually performed in the object–meter system. This idea is expressed in the most compact form by introducing the *entangling* measurement transformation [11]. In limiting cases, this transformation is reduced to either the standard quantum measurement with a classical representation of the result or to a unitary transformation in the object–meter system, for which the result of measurement can be presented in the purely quantum entanglement form establishing the 100%-correlation between the singled-out variables of the object and of the meter.

The preferred position of entangling measurements among a large number of other transformations is due to the fact that such measurements make it possible to distribute an object among a large number of quantum systems without disturbing the singled-out set of states $|k\rangle_A$. In the limiting case of a purely coherent measurement, such a transformation for a fixed initial state $|0\rangle_B$ of the meter represents the initial basis states $|k\rangle_A$ of the object in terms of doubling states $|k\rangle_A|k\rangle_B$ of the object–meter system, while arbitrary linear combinations $\sum_k c_k |k\rangle_A$ are represented by analogous linear combinations of doubling states. On the other hand, coherent quantum relations between wave functions of the object are now transferred to the same relations in the object–meter system (i.e., such relations are not conserved in the object since essentially quantum information cannot be doubled). A standard quantum measurement is accompanied by additional loss of the initial phase relations. Thus, the concept of quantum measurements is associated not with the classical nature of the physical representation of extracted information, but with a more primary concept of classicism, viz., the possibility of its unlimited reproduction. Such an approach makes it possible to consider the processes of extraction and transformation of quantum information from unified positions suitable for classical as well as quantum-mechanical instruments.

In addition to the concept of entangling measurement as the establishment of one-to-one correspondence between sets of classically compatible states of the object–meter quantum systems, we can also introduce generalized types of measurement analogously to classical instruments. It was shown in [1] that we can single out two main versions of measurement as the establishment of correspondence with the help of the classical information index: (i) fuzzy measurement as the establishment of the correspondence between a classically compatible set of states of the object and an incompletely distinguishable set of quantum states of the meter (in fact, this is the simplest special case of so-called "fuzzy" measurements [12, 13] and (ii) a partially destructive quantum measurement as the establishment of the correspondence between an incompletely distinguishable set of states of the object and a classically compatible set of states of the meter.

Examples of such generalized measurements of two-level quantum systems were considered in [1]. However, it would also be interesting to consider measurements of this type for systems with infinitely dimensional spaces of states, when dynamic variables with continuous values are to be measured. In particular, measurements with the help of a classically compatible set of coordinates of two oscillator (e.g., photon) modes playing the role of a meter of two classical incompatible quadrature components of a mode playing the role of the object are of special interest in view of the existence of a natural simple correspondence between the sets of canonical variables in the object–meter system. In this case, we can expect that the simplicity of the correspondence being established in such measurements is the ground for its possible experimental implementation (e.g., using the methods of nonlinear optics). In addition, measurements of this type reveal the fundamental properties of the process of extraction of information from quantum systems (beginning from quantum limitations on the potential precision of measurements and ending with quantitative characteristics of dequantization noise) in its most general and concentrated form.

2. TRANSFORMATION OF CONTINUOUS MEASUREMENT OF A CANONICAL SET OF QUANTUM VARIABLES

The simplest and most fundamental example of measurement of continuous variables is the measurement of electron coordinate $\hat{q}$, which is considered in almost all textbooks of quantum mechanics. It is well known that only this half of the complete set of dynamic canonical variables $\hat{X}^T = (\hat{q}, \hat{p})$ (T is the symbol of matrix transposition) of a quantum system with a single degree of freedom in quantum theory that can be measured exactly (in contrast to the classical theory).

To establish a more complete relation between the quantum and classical theories, we must also consider the measurement that leads to the minimal possible error for both canonical variables $\hat{X}$ and ensures their precise measurement in the semiclassical limit $\hbar \longrightarrow 0$. Measurements of this type are not only possible [9], but also optimal under corresponding conditions [10, 14, 15]. Such measurements are based on the simple fact that a quantum system with a doubled number of degrees of freedom contains the same number of simultaneously measurable coordinate operators as their number in the complete set of canonical variables of the object. This type of measurements naturally emerge in the approach [1] including the generalization to essentially quantum-mechanical meters based on the concept of entangling measurement and on the rejection of the requirement concerning a nondestructive form of the measurement at the quantum level of accuracy.

2.1. Superoperator of a Partly Destructive Measurement

In compact form, a measurement can be described by a superoperator mapping density matrices $\hat{\rho}_A$ of the object into density matrices $\hat{\rho}_{AB}$ of the object–meter system. Since it is expedient to assume that the initial state of the meter is fixed in the form of a pure state $\hat{\rho}_B^0 = |0\rangle_B\langle 0|_B$, the complete transformation in the object–meter system, which can be realized in various equivalent ways, is hence immaterial.

The superoperator describing in the general case a partly destructive entangling measurement of a set of nonorthogonal states $|\alpha)_A$ of the object with the help of a set of orthogonal (i.e., classically distinguishable) states $|\alpha\rangle_B$ of the meter, has the form

$$\mathcal{M} = \sum_{\alpha\beta} R_{\alpha\beta}\sqrt{\nu_\alpha\nu_\beta}(\alpha|_A \odot |\beta)_A|\alpha)_A|\alpha\rangle_B\langle\beta|_B(\beta|_A. \quad (1)$$

Here, $R_{\alpha\beta}$ is the entanglement matrix satisfying the conditions $R = R^+$, $R_{\alpha\alpha} \equiv 1$, $R \geqq 0$ and controlling the degree of dequantization of the results of measurement due to the interaction of the meter with dephasing system D (in the simplest case of entanglement mapped by an entanglement matrix of the form $R_{\alpha\beta} = (\alpha|\beta)_D$ [16]), and the set of positive numbers ν_α describes the degree to which the corresponding states $|\alpha)_A$ are represented

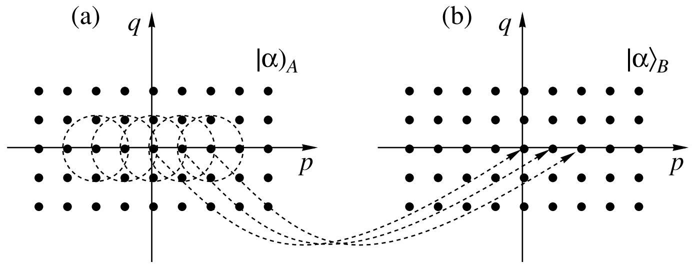

Fig. 1. Mapping of elementary states in the course of a generalized measurement. The quantum indeterminacy in the values of coordinates and momenta, corresponding to states $|\alpha)_A$ of the object being measured, which are mapped by the classically distinguishable states $|\alpha\rangle_B$ of the meter, is indicated.

and defines the corresponding positive operator-valued measure (POVM)

$$\hat{E}_\alpha = \nu_\alpha|\alpha)_A(\alpha|_A, \quad \sum \hat{E}_\alpha = \hat{I}_A. \tag{2}$$

The limiting cases $R_{\alpha\beta} = \delta_{\alpha\beta}$ and $R_{\alpha\beta} = 1$ describe the standard projective measurement corresponding to complete dephasing of the states being measured, as well as the reversible isometric map of the Hilbert space of the object into the space of states of the object–device corresponding to the absence of dephasing.

In the case of an orthogonal set of states $|\alpha)_A = |\alpha\rangle_A$, this superoperator transforms the initial states of the given type into states $|\alpha)_A|\alpha\rangle_B$ of the object–meter system (i.e., is nondestructive). On the other hand, the nonorthogonality of the states being measured makes their preservation impossible and accordingly, assigned to the partly destructive nature of measurement. It can easily be seen that the subalgebras of classically compatible states and of dynamic variables corresponding to them are not distorted asymptotically (in the presence of the corresponding small nonclassicality parameter); i.e., the given measurement considered at classical level is precise and nondestructive.

For simplicity, let us confine our analysis to a one-dimensional (unimodal) object, which corresponds to the following commutator in canonic variables:

$$\hat{X}\hat{X}^T - (\hat{X}\hat{X}^T)^T = C = \begin{pmatrix} 0 & i\hbar \\ -i\hbar & 0 \end{pmatrix}.$$

Considering a uniform mesh with points $X_\alpha = (q_\alpha, p_\alpha)$ on a plane with coordinates q, p, we choose as measurable states $|\alpha)_A$ in expression (1) the states obtained by displacement transformation $U(X_\alpha) = \exp(-X_\alpha^T C^{-1}\hat{X})$ of initial state $|0)_A$:

$$|\alpha)_A = U(X_\alpha)|0)_A; \tag{3}$$

in this case, $U^{-1}(X_\alpha)\hat{X}\,U(X_\alpha) = \hat{X} + X_\alpha$. As the basis state $|0)_A$, we can choose a state characterized by the projector

$$|0)_A(0|_A = \lim_{\varepsilon \to 0} e^{\Gamma - \frac{1}{\varepsilon}\hat{X}^T Q_0 \hat{X}} \tag{4}$$

obtained as a limit of the Gaussian density matrix with a quadratic form defining in the general case a compressed state with a rotated dispersion ellipse.

In this case, the multiplicity of states must be chosen in the form

$$\nu_\alpha = \frac{dX_\alpha}{2\pi\hbar}, \tag{5}$$

where dX_α describes the cell volume per state $|\alpha)_A$. It is well known for $|\alpha)_A$ in the form of coherent states [10, 17], the set of operators (2) in this case indeed corresponds to the POVM in the limit $dX_\alpha \longrightarrow 0$ and, hence, transformation (1) defines the corresponding generalized measurement with the results of measurement in the form of classically distinguishable (orthogonal) states $|\alpha\rangle_B$. Thus, the transformation of continuous measurement of a set of canonical quantum variables is defined as a limiting transition to an infinitely dense mesh. The corresponding mapping is illustrated in the figure.

To establish the correspondence between variables $\hat{X}$ of the object and corresponding commuting variables $\hat{X}_B$ on the basis of the mapping of quantum states described above, we must take into account the representation

$$\hat{X}_B = \sum_\alpha X_\alpha|\alpha\rangle_B\langle\alpha|_B \tag{6}$$

for discrete readings of the meter. In the limit of an infinitely dense mesh of states, this expression can be described by generalized matrix elements of the form $(\hat{X}_B)_{X'X''} = \int X\delta(X' - X)\delta(X'' - X)dX$. In fact, measurement (1) for the above characteristics differs from those considered in [9, 10, 12, 14] only in one new feature, viz., essentially quantum-mechanical representation of output data in the general case, which is manifested in the presence of entanglement matrix $R_{\alpha\beta}$ differing from unity.

The measurement considered here is entirely nonselective to translations in the phase space of canonic variables $\hat{X}$; in this respect, it is analogous to a nonselective partly destructive measurement in a two-level system [1]. However, in contrast to the latter measurement, which is invariant to any unitary transformation of the initial state, it is still selective to, for example, operation of compression or rotation of the dissipation ellipse of the initial state, which are controlled by quadratic form Q_0.

2.2. Quantitative Characteristics of Measurement

First, we calculate the mean square error of measurement for initial state $\hat{\rho}_A = |\alpha_0)(\alpha_0|$,

$$\Delta_S = \mathrm{Tr}(\hat{X}_B - X_{\alpha_0})^T Q(\hat{X}_B - X_{\alpha_0})\mathcal{M}\hat{\rho}_A, \qquad (7)$$

for parameters of this state, which characterizes the "semiclassical" properties of the measuring channel. Here, Q is the quadratic form matrix taking into account the contributions from the components of the vector being measured. In such an approach, we in fact assess the informativity of the semiclassical information channel $\alpha_0 \longrightarrow \hat{\rho}_B = \mathrm{Tr}_A\mathcal{M}\hat{\rho}_A$ realized with the help of measurement $\mathcal{M}$. Another measure of informativity is the Holevo information [18].

Substituting the Gaussian density matrix in the form $\hat{\rho}_A = \exp[\Gamma - (\hat{X} - X_{\alpha_0})^T Q_0(\hat{X} - X_{\alpha_0})/\varepsilon]$ for $\varepsilon \longrightarrow 0$ as the initial state into formula (1) (this matrix corresponds to vacuum $|0)$ coherently displaced by transformation $U(X_{\alpha_0})$, $\hat{X}^T Q_0 \hat{X}|0) = \frac{1}{2}\mathrm{Tr}|Q_0 C|\,|0)$) and using the standard technique for transforming Gaussian operators [19], we obtain the Gaussian density with scattering matrix $2K_0$ for distribution X_α, which is twice as large as the vacuum matrix,

$$\nu_\alpha|(\alpha|\alpha_0)_A|^2 \longrightarrow \frac{dX_\alpha}{\det(4\pi K_0)} e^{-\frac{1}{4}X_\alpha^T K_0^{-1} X_\alpha},$$

$$K_0 = \frac{1}{2} C \,\mathrm{sgn}\, Q_0 C$$

and, accordingly,

$$\Delta_S = \sum (X_\alpha - X_{\alpha_0})^T$$

$$\times Q(X_\alpha - X_{\alpha_0})\nu_\alpha|(\alpha|\alpha_0)_A|^2 = 2\mathrm{Tr}QK_0.$$

For quadratic forms with matrices $Q = Q_0$, which correspond to Hamiltonian $\hat{H} = \hat{p}^2/2m + m\omega^2\hat{q}^2/2$ of the same harmonic oscillator, this expression leads to the result $\Delta_S = \mathrm{Tr}|QC| = \hbar\omega$ that implicitly appears in [10]; i.e., quadratic indeterminacy is equal to a doubled energy of vacuum fluctuations. The doubling is of fundamental importance and is due to the addition of dequantization noise contained in variables $\hat{X}_B$ and independent of these fluctuations to the initial quantum fluctuations of variables $\hat{X}$ in state $|\alpha_0)_A$.

The mean standard error for canonical operators $\hat{X}$ of the object,

$$\Delta_Q = \mathrm{Tr}(\hat{X}_B - \hat{X})^T Q(\hat{X}_B - \hat{X})\mathcal{M}\hat{\rho}_A, \qquad (8)$$

which is observed after the measurement has been performed, characterizes the measurement as a quantum channel providing information on classically incompatible quantum variables of the object. Analogous calculation for state $\hat{\rho}_A|\alpha_0)(\alpha_0|$ of the object taking into account the invariance of difference operator $\hat{X}_B - \hat{X}$ and its diagonal form in states $|\alpha\rangle_B$ gives

$$\Delta_Q = \sum \nu_\alpha(\alpha|\hat{\rho}_A|\alpha)\langle\alpha|_B(\alpha|_A(\hat{X}_B - \hat{X})^T$$

$$\times Q(\hat{X}_B - \hat{X})|\alpha)_A|\alpha\rangle_B$$

$$= (0|_A \hat{X}^T Q \hat{X}|0)_A = \mathrm{Tr}QK_0,$$

which is half the value of the error for the meter as a semiclassical channel.

This error is equal to the minimal possible error compatible with noncommutativity of operators $\hat{X}$ and, accordingly, $\hat{X}_B - \hat{X}$. Thus, we can state that measurement (1) not only provides an effective estimate for parameters of a coherent Gaussian state, but also can be treated as a method for obtaining an optimal estimate of the canonical variables $\hat{X}$ themselves of a quantum object, irrespective of the form of entanglement matrix R and, hence, the degree of coherence of the transformation being carried out.

The partly destructive form of measurement (1) is manifested in the change in the initial state upon mapping $\hat{\rho}_A \longrightarrow \hat{\rho}'_A = \mathcal{M}_A\hat{\rho}_A$, where $\mathcal{M}_A = \mathrm{Tr}_B\mathcal{M}$ is the superoperator of transformation of states of the object irrespective of the result of measurement. In the case of a two-level object, the superoperator can be expressed in terms of the vectors of quasi-spin operators $\hat{\boldsymbol{\sigma}}$ in the form $\mathcal{M}_A + \frac{1}{3}\hat{\boldsymbol{\sigma}} \odot \hat{\boldsymbol{\sigma}}$ [1]. The destructive form of the measurement for initial state $\hat{\rho}_A = \frac{1}{2}(\hat{I} = \mathbf{s}\hat{\boldsymbol{\sigma}})$ is manifested in its depolarization $\mathbf{s} \longrightarrow \frac{1}{3}\mathbf{s}$. In the case under investigation, we have $\mathcal{M}_A = \int \frac{dX_\alpha}{2\pi\hbar} |\alpha)_A(\alpha|_A(\alpha|_A \odot |\alpha)_A$; for initial state $\rho_A = |0)_A(0|_A$, we obtain a distorted state in the form

$$\hat{\rho}'_A = \int \frac{dX_\alpha}{2\pi\hbar} |(\alpha|0)_A|^2 |\alpha)_A(\alpha|_A = \int \frac{dX_\alpha}{\det^{1/2}(4\pi K_0)}$$

$$\times e^{-\frac{1}{4}X_\alpha^T K_0^{-1} X_\alpha} |\alpha)_A(\alpha|_A = \frac{1}{\det^{1/2}(6\pi K_0)} \mathcal{W} e^{-\frac{1}{6}\hat{X}^T K_0^{-1}\hat{X}},$$

where symbol $\mathcal{W}$ indicates Wigner ordering symmetric in operators $\hat{q}$ and $\hat{p}$. The correlation matrix corresponding to this Gaussian matrix is $3K_0$. It can easily be seen that perturbation introduced by the measurement can be reduced to tripling the scattering matrix ($K_0 \longrightarrow 3K_0$) for displaced initial states of the form $|\alpha_0)_A$ also. In the case of the arbitrary Gaussian states $\hat{\rho}_A$ with correlation matrix K and mean value of $X_0 = \langle \hat{X} \rangle$, the distorted state is characterized by the change in correlation matrix $K \longrightarrow K + 2K_0$ with the conserved mean value in accordance with the following relation that holds for the correlation Gaussian density matrices:

$$\sum \nu_\alpha (\alpha|\hat{\rho}_A|\alpha)|\alpha)(\alpha| \longrightarrow K + 2K_0.$$

For a purely coherent state, $\hat{\rho}_A = |\alpha_0)(\alpha_0|$ corresponds to $3K_0$.

3. DEQUANTIZATION NOISE

Taking into account the isometric nature of transformation $\mathcal{M}$ for $R_{\alpha\beta} \equiv 1$, we obtain identity $\mathrm{Tr} f[\mathcal{M}\hat{\rho}_A] = \mathrm{Tr} f[\hat{\rho}_A]$ for an arbitrary scalar function f, which shows that the entropy of the object–meter system after such a completely coherent measurement coincides with the initial entropy of the object and is zero in the case of its pure state corresponding to density matrix $K = K_0$: $S_{AB}[K]|_{K = K_0} = 0$. Since states $|\alpha)_A$ are not orthogonal, these states are coded in a complex combination of orthogonal two-component states $|\alpha)_A|\alpha\rangle_B$. The explicit form of mapping $A \longrightarrow A + B$ of quantum information is defined by corresponding expressions for overfilled basis

$$|\alpha)_A \longrightarrow \sum_\beta \sqrt{\nu_\beta} (\beta|\alpha)_A |\beta)_A |\beta\rangle_B.$$

As we pass to the standard measurement, two-partite states $|\alpha)_A|\alpha\rangle_B$ acquire an additional degree of freedom, which corresponds to the independent orthogonal states $|\alpha\rangle_D$ of the dephasing subsystem (shot noise). Consequently, the entropy of the object–meter system reflects independent dequantization fluctuations and, hence, increases indefinitely upon a decrease in the mesh volume of the measuring net.

To calculate shot noise introduced by the dephasing subsystem upon partial dequantization of the result of measurement, we consider the joint density matrix

$$\rho^{AB}_{\alpha\beta} = R_{\alpha\beta} \rho^A_{\alpha\beta} \sqrt{\nu_\alpha}\sqrt{\nu_\beta}. \quad (9)$$

For $R_{\alpha\beta} \equiv 1$, this matrix defines an isometric map of the input density matrix, and their entropies coincide, which corresponds to the absence of dequantization noise. On the other hand, inequality $R_{\alpha\beta} \neq 1$ describes the selection of classically compatible states $|\alpha\rangle_B$ of the meter coherently coupled for $R_{\alpha\beta} \equiv 1$ by the dephasing subsystem. The phase indeterminacy in the object–meter system introduced in this way violates the coherence of mapping and, hence, leads to dequantization of the resultant set of states. The only measure of dequantization noise introduced into the system is the corresponding increment of entropy as compared to a completely coherent measurement:

$$\Delta S_d = S[\hat{\rho}_{AB}] - S[\hat{\rho}_{AB}]|_{R \equiv 1} = (S[\hat{\rho}_{AB}] - S[\hat{\rho}_A]). \quad (10)$$

Let us calculate ΔS_d for *semiclassical* density matrices $\hat{\rho}_A$ of the object, which can be defined as follows:

$$\rho^A_{\alpha\beta} \approx dX_\alpha w(X_\alpha)(\alpha|\beta), \quad (11)$$

where $w(X_\alpha)$ defines the Wigner density of probability distribution, which slowly varies on the scale of an elementary phase cell with a volume of $2\pi\hbar$. The corresponding entropy is calculated taking into account the relation $(\rho^A_{\alpha\beta})^n = [w(X_\alpha)2\pi\hbar]^{n-1} w(X_\alpha) dX_\alpha (\alpha|\beta)$ following from the completeness condition (2) of states $|\alpha)$ and expression (5) and is defined by the following semiclassical expression, which is well known in statistical physics:

$$S[\hat{\rho}_A] = -\int (\log[w(X)2\pi\hbar] w(X)) dX. \quad (12)$$

Here, phase volume $2\pi\hbar$ maps the quantum indistinguishability of states α, which is taken into account by factor $(\alpha|\beta)$ in representation (11).

Calculating entropy $S[\hat{\rho}_{AB}]$ taking into account dephasing $R_{\alpha\beta}$, we consider the case of intense dephasing, in which off-diagonal matrix elements $R_{\alpha\beta}$ decay as a function of difference $X_\alpha - X_\beta$ at a much higher rate than on the scale of elementary phase volume $2\pi\hbar$ corresponding to quantum nonorthogonality factor $(\alpha|\beta)$. In this case, we can disregard the last factor in the expression for joint density matrix $\hat{\rho}_{AB}$ and proceed from the expression

$$\rho^{AB}_{\alpha\beta} \approx dX_\alpha w(X_\alpha) R_{\alpha\beta}.$$

This approximation enables us to obtain an asymptotic estimate $S[\hat{\rho}_{AB}] = -\int \log[w(X)v_c]w(X)dX$, which is analogous to the above estimate, but contains *coherence volume* v_c corresponding to $R_{\alpha\beta}$ instead of the elementary quantum volume. This estimate corresponds to the following expression for the entropy increment,

$$\Delta S_d = \int \log\left(\frac{2\pi\hbar}{v_c}\right) w(X)dX, \quad (13)$$

which is valid for $v_c \ll 2\pi\hbar$ and takes into account the possible dependence of the coherence volume v_c on phase variables X in the general case. This heuristic estimate becomes rigorous if we assume a special structure of entanglement matrix $R_{\alpha\beta} = (\alpha|\beta)_v$, where the scalar product reproduces the same product for coherent states, but its relation with operator algebra differs in that the Planck constant $\hbar$ is replaced by $v_c/2\pi$. This assumption does not contradict in any way the fundamentals of quantum theory, since the only conditions met by matrix $R_{\alpha\beta}$ is its positive value and equality of diagonal elements to unity.

Thus, dequantization of measured information is accompanied by the introduction of dequantization noise; in the limit of standard measurement, the contribution of such noise to the total quantum entropy of the object–meter system tends to infinity. However, this noise does not affect the precision of measurement, since readings X_α of the meter do not asymptotically distinguish potentially distinguishable states $|\alpha\rangle_B$ in an infinitely small range of values of $X_\alpha \in V$, which corresponds to an infinitely large number of coherence volumes $v_c \longrightarrow 0$. The error of measurement is limited in this case only by the intrinsic quantum indeterminacy of variables $\hat{X}$ being measured.

4. CONCLUSIONS

Thus, as applied to the measurement of continuous variables, general definition of partly destructive entangling quantum measurement [1] for a canonical set of variables being measured gives a generalization of measurement of the noncommuting quantum variables discussed earlier [9, 10, 14, 15] to the case of a quantum-mechanical meter extracting resultant information, not necessarily in classical form, but in a form which permits its unlimited reproduction.

In the type of measurement considered here, the distortion of the initial states of the object is associated with the representation of quantum-mechanically incompatible variables and corresponding sets of nonorthogonal states in classical form.

Dequantization of output information leads to introduction of dequantization noise associated with the interaction of the object and the meter with the dephasing subsystem. In the limit of standard completely dequantizing measurement, this noise leads to an infinitely large increase in the total quantum entropy in the object–meter system without affecting the quality of measurement.

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